NALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE

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ABSTRACT
The present study is concerned with upset forging of arbitrarily-shaped prismatic blocks which is characterized by three-dimensional deformation. From the proposed velocity field, the upper-bound load and deformed configuration are determined by minimizing the total power consumption with respect to some chosen parameters; such as the proposed method of analysis in this work can be used for prediction of forging load and deformation in upset forging of arbitrarily-shaped prismatic blocks. Closed-die forging of toothed disk is investigated using the slab method technique. The tooth regions are approximated by prismatic rectangular sections. The velocity field comprising three-unit deformation regions is used. A constant frictional stress between work-piece and forging die is assumed. The average punch pressure normalized by the flow stress of the billet material is determined theoretically and compared with the present experimental results. The experimental work is carried out on a commercial pure Aluminum (Al 1100) at room temperature. The forging process is carried out using one die geometry without using any additional blocker dies. The theoretical predictions of forging pressures and deformation configurations are in good agreement with the experimental results. The experimental work proves that the analysis is good within approximately 36% of error according to many parameters.

Keywords: Closed Die forging; cold forming; Three Dimentional Slab method; Spur Gear

II. INTRODUCTION
Forging is the process of deforming a metal in order to shape a product into a given desired configuration. The prediction of metal flow or geometric change at the free surface of a workpiece is very important. The main factors affecting metal flow are friction, strength of the metal being formed, and the geometry of the tools and the workpiece.

Procedures for the analysis of cylindrical upsetting have been developed by various theoretical methods such as the slip-line method, the upper bound method, and the finite element method ... etc. It is, however, only recently that the upset forging of non-circular blocks have been analyzed.

With the use of closed-die forging, complex shapes and heavy reductions can be made within closer dimensional tolerances which are usually feasible with open dies. Closed-die forging is adaptable to high-volume production improved structure, and good mechanical properties and surface finish. In closed-die forging a material must satisfy two basic requirements:

a. The material strength (or flow stress) must be low so that die pressures are kept within the capabilities of practical die materials and constructions.
b. The capability of the material to deform without failure (its forgeability) must allow the desired amount of deformation for a given material.

Both the flow stress and the forgeability are influenced by the metallurgical characteristics of the billet material and the forging parameters such as temperature, strain, strain rate and stress.
In the most practical closed-die forging operations, the temperature of the workpiece material is higher than that of the dies. Material flow and die filling are largely determined by the resistance and the ability of the forging material to flow, i.e., flow stress and forgeability. Flow stress represents the resistance of a material to plastic deformation while forgeability is the ability of a material to deform without failure regardless of the magnitude of load and stress required for deformation.

Friction generally influences metal flow, pressure distribution, load, and energy requirements in closed-die forging since there is a considerable effect of friction on forgeability. Therefore, the friction problem must be treated carefully by using the appropriate lubricant.

Precision forged gears are made from billets have almost the exact volume of the material required for the final size of the gears. No allowance is made for flash formation. The development of precision forging gear processes has been an area of increasing activity in recent years. The analysis of corner filling characteristics in precision forging considering the effect of difference in workpiece geometry and lubrication showed that the velocity field which leads to the best load prediction is not that which most closely describes changes of geometry in a billet. Kim et al [4] proposed some appropriate velocity fields for the upsetting analysis of three dimensional forging of arbitrarily-shaped prismatic blocks. The analysis showed a good agreement of forging load with their experimental results. Forging processes for heavy ingots by finite element method had been analyzed by Sun [5] who proposed the optimum geometrical parameters for the forging process. Large deformation behavior of Aluminum and low carbon steel short cylinders loaded axially in simple compression without using any lubricant was examined by Gupta and Shah [6]. Their results revealed that the profile of a deforming specimen can be approximated by an arc of a circle only after the onset of folding.

Forging of spur gear forms and closed die forging of gear-like elements using the upper bound technique have been analyzed [7,8]. In their analysis the assumption of no axial velocity in the tooth region imposes severe limitations on the validity of the analysis.

In the present work, the upper bound technique and slab method have been used to analyze the closed-die forging of a toothed part. Numerical calculations have been done to study the effect of process and material variables on forging load estimation. To determine the validity of the present analysis, an experimental program is carried out on commercially pure aluminum (Al 1100) billet to forge a toothed part having 12 teeth.

**III. RELATED WORKS**

The technology developed includes two steps in the forging process. Well-shaped products are forged successfully using a lower forging pressure than that of conventional forging. The accuracy of the forged spur gear obtained by the new precision forging technology is set nearly equal to that of a cut spur gear of the fourth and the fifth classes in the Korean industrial standard. [J.C Choia 1999]. Forging spur gear forms in completely closed cavity dies is investigated by means of an upper bound analysis. A velocity field comprising three unit deformation regions is proposed. The tooth regions are approximated by prismatic rectangular sections. The effects of root diameter, number of teeth and workpiece/die interface friction, on flow and forging pressures, are determined.

Forging pressure without friction is independent of root diameter but increases with the number of teeth. In the presence of friction forging pressure increases with reducing root diameter. [N.A. Abdul 1986]. The filling of corners in flashless forging of cylindrical shapes is considered. Experiments with commercially pure aluminum were carried out at room temperature and the effects of different workpiece geometries and lubrication on corner filling were observed. Three different upper bound solutions are applied to predict tool loads and metal flow. The extent of agreement between each of these and experimental results depends
on billet geometry and boundary conditions. It is demonstrated that the velocity field which leads to the best load prediction is not that which most closely describes changes of geometry in a billet. [A.O.A. Ibhadode 1988].

A new technique to form spur gear from hollow billet was proposed, that is performing divided flow region and then final forging by the relief hole. At the same time 3D-FEM simulation of the whole process is performed using DEFORM-3D TM software. The results show that this new technique has many advantages over the conventional one, and gives a theory foundation for practical industry production. [XIA Shi-sheng, 2003].

The manufacture of bevel and cylindrical spur gears by the means of applying hot or cold bulk forming processes is a quite widespread production method due to its well-known basic advantages, such as material and time cost reduction and the increased strength of the teeth. However, the associated process planning and tool design are more complicated compared with those of other conventional forging or rolling technologies. [B.I Tomov 1999].

Insufficient corner filling is one of the main disadvantages of conventional cold closed-die forging of spur gears. To guarantee the dimension accuracy of spur gears, how to improve the filling condition is very important. Because the die shape is one of the most important factors in forming, three design schemes with different die shape are researched. A corresponding experiment is done, which is mainly utilized for supporting and validating the numerical simulation and theoretical investigation. [Chengliang Hua 2007]. Using the present model, various effects of forming parameter such as the friction factor, reduction, number of teeth, etc. upon the non-dimensional forging pressure, forging force and barreling of the spur gear forms and spline were analyzed systematically and the results compared with those of other researcher's analytical and experimental work. [Hung-Hsiou Hsu 2002]. The application of tool design expertise in conjunction with the virtual prototyping modeling techniques made possible the development of a tool system for performing the experiments. The experimental work is mainly utilized for supporting and validating the theoretical investigation but a later stage can be further developed in order to put the proposed forging concept on an industrial basis. [M.L Alvesa 2001]. The forging of spur gears has been investigated by means of upper-bound analysis. A kinematically admissible velocity field for the forging of spur gears has been newly proposed; especially, a neutral surface has been introduced into the forging of gears by using hollow billets with a flat punch. The half pitch of the gear has been divided into seven deformation regions. By using the kinematically admissible velocity field, the power requirements and suitable conditions for the forging of spur gears were successfully calculated by a numerical method. [Jongung Choia, 2000].

IV. METHODOLOGIES/THEORETICAL SETUP

Slab Method Analysis

A circular punch is used for forging a cylindrical billet placed in a die which has spaces for teeth on its periphery. The punch compresses the billet axially and as a consequence the material flow outward into the teeth spaces in the radial directions. In the initial stage of the forging process it is assumed that the billet closely fits the die which has a diameter equal to the dedendum diameter and that the reduction in height allows the material to flow in the teeth cavities. The deformation pattern of the material is considered as axisymmetric forging process for a billet which has a diameter equal to the root diameter of the toothed disk. The tooth formation is considered to take place as plane strain forging process. Integration for plane strain forging of teeth and axisymmetric forging of billet gives the complete solution of spur gear forging problem.

The cylindrical and Cartesian coordinate systems are used to carry out the analysis. The cylindrical coordinate \((r, \theta, z)\) is located at mid.-point of the billet while the Cartesian coordinate \((x, y, z)\) is located at the point of junction of tooth with the billet as shown in Fig.(1).
The analysis proceeds as follows:
Consider first the equilibrium of forces in the x-direction:

\[ \sigma_x \, w - 2\mu \, \sigma_y \, w\, dx - 2\mu \, \sigma_z \, h\, dx - (\sigma_x + d\sigma_x) \, w = 0 \]

For plane strain condition:

\[ \frac{d\sigma_x}{dx} + 2\mu \left( \sigma_y \, \frac{\sigma_x}{h} + \sigma_z \, \frac{\sigma_x}{w} \right) = 0 \]

Substituting in Eq. (1)

\[ \frac{d\sigma_x}{dx} + 2\mu \left( \sigma_y \, \frac{\sigma_x}{h} + \sigma_z \, \frac{\sigma_x}{w} \right) = 0 \]

\[ = \frac{1}{\sqrt{2}} \left[ \sigma_x^2 - 2\sigma_x \sigma_y + \sigma_y^2 + \sigma_x^2 - 2\sigma_x \sigma_y + \sigma_y^2 + \sigma_x^2 - 2\sigma_x \sigma_z + \sigma_z^2 \right]^{1/2} \]

\[ = \frac{\sqrt{3}}{\sqrt{2}} \left[ \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - (\sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z) \right]^{1/2} \]

\[ = \left[ \frac{\sigma_x^2}{2} + \frac{\sigma_y^2}{2} + \frac{\sigma_z^2}{2} - \frac{\sigma_x \sigma_y}{2} - \frac{\sigma_x \sigma_z}{2} - \frac{\sigma_y \sigma_z}{2} \right]^{1/2} \]

\[ = \frac{3}{4} \sigma_x^2 + \frac{3}{4} \sigma_y^2 - \frac{3}{2} \sigma_x \sigma_y \]

\[ = \frac{\sqrt{3}}{2} \left( \sigma_x^2 + \sigma_y^2 - 2\sigma_x \sigma_y \right)^{1/2} \]

\[ = \frac{\sqrt{3}}{2} \left[ (\sigma_x - \sigma_y)^2 \right]^{1/2} \]

Substituting Eq. (4) into Eq. (3)

\[ \frac{d\sigma_x}{dx} + \frac{\mu \, \sigma_x}{w} + (\sigma_x - \frac{2\sigma}{\sqrt{3}}) \left( \frac{2\mu}{\sqrt{3}} + \frac{\mu}{w} \right) = 0 \]

\[ \frac{d\sigma_x}{dx} + \frac{\mu \, \sigma_x}{w} + \frac{2\sigma_x \mu}{h} + \frac{\sigma_x \mu}{w} - \frac{4\sigma\mu}{3h} - \frac{2\sigma\mu}{w} = 0 \]

Consider \( \sigma \) is not a function of \( x \), then,
\[
\frac{d\sigma_x}{dx} + C_1 \sigma_x = C_2
\]

Where the constants \( C_1 \) and \( C_2 \) are:
\[
C_1 = 2\mu \left( \frac{1}{w} + \frac{1}{h} \right)
\]
\[
C_2 = \left( \frac{4\mu}{\sqrt{3} h} + \frac{2\mu}{\sqrt{3} w} \right) \bar{\sigma}
\]

The solution of the differential equation 5 is:
\[
\sigma = \frac{C_2}{C_1} + De^{-C_1x}
\]

Where \( D \) is a constant can be determined from the boundary conditions as follows:
\[
\alpha = x = \ell \Rightarrow \sigma_x = 0
\]

Therefore
\[
D = \left( \frac{C_2}{C_1} \right) e^{\frac{C_2}{C_1} \ell}
\]

Sub. Eq. (7) into (6)
\[
\alpha x = \frac{C_2}{C_1} + \left( -\frac{C_2}{C_1} e^{C_1x} \right)
\]
\[
= \frac{C_2}{C_1} - \frac{C_2}{C_1} e^{C_1(\ell-x)}
\]
\[
\alpha x = \frac{C_2}{C_1} \left[ 1 - e^{C_1(\ell-x)} \right]
\]

From Eq. 8 and Eq. 4
\[
\frac{2\sigma}{\sqrt{3}} + \sigma_y = \left( \frac{C_2}{C_1} \right) \left[ 1 - e^{C_1(\ell-x)} \right] - \frac{2\bar{\sigma}}{\sqrt{3}}
\]
\[
\sigma_y = \left( \frac{C_2}{C_1} \right) \left[ 1 - e^{C_1(\ell-x)} \right] - \frac{2\sigma}{\sqrt{3}}
\]
\[
\sigma_y = \frac{2\mu}{\sqrt{3}} \bar{\sigma} \left( \frac{2}{h} + \frac{1}{w} \right) \left[ 1 - e^{C_1(\ell-x)} \right] - \frac{2\sigma}{\sqrt{3}}
\]
\[
\sigma_y = \frac{2\mu}{\sqrt{3}} \bar{\sigma} \left[ \frac{1}{h} + \frac{1}{w} \right] \left[ 1 - e^{C_1(\ell-x)} \right] - 1
\]
\[
C = \frac{1}{h} + \frac{1}{2w}
\]
\[
\sigma_y = \frac{2}{\sqrt{3}} \bar{\sigma} \left[ C(1 - e^{C_1(\ell-x)}) - 1 \right]
\]
The load required to forge the teeth at any stage of deformation can be obtained by equating the external load to the internal resistance,

\[ \text{Load} = L_{\text{tooth}} = P_{av} \times (w/\ell)N \]  \hspace{1cm} (10)

\[ L_{\text{tooth}} = N \int_{x_0}^{\ell} \sigma y \ w \ dx \]

\[ L_{\text{tooth}} = N \frac{2}{\sqrt{3}} \sigma w \int_{x_0}^{\ell} \left[ C(1 - e^{c(x-x_0)}) \right] dx \]

\[ = N \frac{2}{\sqrt{3}} \sigma w \int_{x_0}^{\ell} \left[ C(1 - e^{c(x-x_0)}) \right] dx \]

\[ = N \frac{2}{\sqrt{3}} \left[ \ell(C - 1) \right] \]

\[ - C \int_{x_0}^{\ell} e^{c(x-x_0)} \ dx = - C \int_{x_0}^{\ell} (e^{c(x-x_0)} - e^{c\ell}) \ dx \]

\[ = - C e^{c\ell} \int_{x_0}^{\ell} e^{c(x-x_0)} \ dx \]

\[ = - C e^{c\ell} \left[ \frac{-1}{C} (e^{c\ell} - e^{cx_0}) \right] \]

\[ = - C e^{c\ell} \left[ e^{c\ell} - 1 \right] \]

\[ = \frac{C}{C_i} \left[ 1 - e^{c\ell} \right] \]

\[ \therefore L_{\text{tooth}} = \frac{Nw\sigma}{\sqrt{3}} \left[ \ell(C - 1) + C \frac{1}{C_i} - e^{c\ell} \right] \]

From Eq. (10)

\[ P_{av} \ \ell = \frac{L_{\text{tooth}}}{wN} \]

\[ \therefore P_{av} \ \ell = \frac{2}{\sqrt{3}} \sigma \left[ \ell(C - 1) + C \frac{1}{C_i} \left( 1 - e^{c\ell} \right) \right] \]

\[ \frac{P_{av}}{\sigma} \text{tooth} = \frac{2}{\sqrt{3}} \left[ (C - 1) + C \frac{1}{C_i} \ell \left( 1 - e^{c\ell} \right) \right] \]  \hspace{1cm} (11)

Equation (11) gives the normalized average pressure required to forge the material in the tooth region.

Take constant volume concept

\[ \pi w^2 h_s = \pi r^2_1 h + NW/h \]

\[ \pi r^2_1 h_s = h (r^2_1 + NW\ell) \]
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\[ \pi r_2^2 h_o = h(\pi r_2^2 + N\beta \ell) \]
\[ \beta = \frac{2\pi}{2N} \]

\[ \pi r_2^2 h_o = h(\pi r_2^2 + N\frac{\pi}{N} r_2 \ell) \]

\[ w = \beta r_2 \]

\[ \pi r_2^2 h_o = \pi r_2 h(r_2 + \ell) \]
\[ w = \frac{\pi}{N} r_2 \]

\[ r_2 h_e = h(r_2 + \ell) \]

\[ r_2 (h_o - h) = h\ell \]

\[ \ell = r_2 \left( \frac{h}{h} - 1 \right) \]

The tooth length at any time during the forging process
\[ \ell = r_3 - r_2 \]
\[ r_3 = r_2 \left[ \left( \frac{h_o}{h} - 1 \right) + 1 \right] \]

For the axisymmetric part of the gear which bounded by the circle \( r_2 \) the force equilibrium in the \( r \) direction gives:
\[ \frac{d\sigma \cdot r}{dr} + \frac{2\mu}{h} \cdot \sigma \cdot r = \frac{2\mu}{h} \cdot \bar{\sigma} \]  \hspace{1cm} (12)

The solution of this inhomogeneous linear diff. Eq. of the first order is:
\[ \sigma_r = e^{-\frac{2\mu}{h} \ell} \left[ \frac{\sigma}{\bar{\sigma}} e^{\frac{2\mu}{h} \ell} + Q \right] \]  \hspace{1cm} (13)

Where \( Q \) is constant can be determined from the boundary conditions;
\[ (\sigma_r) r = r_2 = (\sigma_x) x = 0 \]

\[ C_2 \left[ 1 - e^{c \ell} \right] = e^{-\frac{2\mu}{h} \ell} \left[ -\frac{2\mu}{\bar{\sigma} \cdot h} + Q \right] \]

\[ C_1 \left[ 1 - e^{c \ell} \right] = -e^{-\frac{2\mu}{h} \ell} + e^{2\mu \bar{\sigma} \cdot h} + e^{-2\mu \bar{\sigma} \cdot h} \cdot Q \]

\[ Q = \frac{C_1}{C_2} \left[ 1 - e^{c \ell} \right] \cdot e^{-\frac{2\mu}{h} \ell} = \frac{C_1}{C_2} \left[ -e^{-\frac{2\mu}{h} \ell} + e^{3\mu \bar{\sigma} \cdot h} \right] \]

\[ Q = e^{-\frac{2\mu \bar{\sigma}}{h}} \left[ C_1 \left[ 1 - e^{c \ell} \right] - \frac{c}{\bar{\sigma}} \right] \]
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Let \[ D_1 = \frac{2\mu}{h} \]

\[ Q = \sigma_0 \left[ \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right] \tag{14} \]

Using the axisymmetric condition

\[ \sigma = \frac{-2\mu}{2h} \left( \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right) + D_1 \left( \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right)^2 \] \tag{15}

\[ \sigma = \sigma_{\theta} \]

These

\[ \sigma = \sigma - \sigma \]

From Eq. (15) - Eq. (16)

\[ \sigma = \sigma_{\theta} \]

Where:

\[ D_2 = \frac{2\mu}{h} \]

The forging load for axisymmetric part of the year is obtained as follows

\[ L_{axi} = \int_0^{2\pi} 2\pi r \sigma dr \]

\[ = P_{av.} \pi r_{axi}^2 \]

\[ \therefore \text{Load}_{axi} = \int_0^{2\pi} 2\pi e (D_1 - D_2 r^2) \left[ \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right] r dr \]

\[ = 2\pi e \left[ \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right] \int_0^{2\pi} (D_1 - D_2 r^2) r dr \]

\[ \therefore L_{axi} = 2\pi e \left[ \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right] D_1 \left( e - D_2 r^2 \left( \frac{r_2}{D_2} + \frac{1}{D_2} \right) \right) + 1 \]

\[ P_{ar.} \pi r_{axi}^2 = 2\pi e \left[ \frac{C_2}{C_1} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right] D_1 \left( e - D_2 r^2 \left( \frac{r_2}{D_2} + \frac{1}{D_2} \right) \right) \]

\[ \frac{C_2}{C_1} = \frac{2\sigma_0}{h} + \frac{1}{h} \]

\[ P_{av.} r_{axi}^2 = 2\pi e \left[ \frac{2\sigma_0}{\sqrt{3}} (1 - \frac{c_i^\ell}{\sigma}) - \frac{\sigma}{\sigma} \right] D_1 \]

\[ r_{axi}^2 (D_1 - D_2 r^2) \]

\[ r_o e \left[ D_1 - D_2 r^2 \right] \]
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\[
\begin{align*}
D_1 &= e \left[ -\frac{D_2 r}{D_2} + \int_{0}^{r} \frac{r}{D_2} \, dr \right]_{2}^{1} \\
&= -e_1 \left[ r e^{-D_2 r} - D_2 r \right]_{0}^{r} \\
&= e D_1 \left[ \frac{t_2 e^{-D_2 t_2}}{D_2} + \left( e^{-D_2 t_2} - \frac{1}{D_2^2} \right) \right] \\
&= e D_1 \left( e^{-D_2 t_2} \left( \frac{t_2}{D_2} + \frac{1}{D_2^2} \right) \right) \\
\]

Where:

\[
I = \left[ -e - D_r r \left( \frac{t_2}{D_2} + \frac{1}{D_2^2} \right) \right]
\]

\[
\left( \frac{P}{\sigma} \right)_{\text{axi}} = \frac{2}{r_1} \left[ \frac{2}{\sqrt{3}} C(1-e^{c}) - \frac{1}{e^{c} I} \right] \tag{18}
\]

Eq. (18) gives the normalized average pressure required to forge the material in the axi-symmetric part of the gear.

By using Eq. (18) Eq. (11)

\[
\left( \frac{P}{\sigma} \right)_{\text{total}} = \left( \frac{P}{\sigma} \right)_{\text{axi}} + \left( \frac{P}{\sigma} \right)_{\text{tooth}} \tag{19}
\]

* Modification of the analysis with the angle \( \Theta \):

\( \Theta = 11.5^\circ \) See Fig. ( )

\( \sigma_w - 2 \mu \sigma_r \), wdx - \( 2 \mu \sigma_y \) \( \cos \theta \) hdx - (\( \sigma_w \) + d\( \sigma_r \) \) wh = 0

For plane strain condition

\[
\frac{d \sigma}{d_x} + \frac{2 \mu \sigma_y}{h} \left( \frac{\sigma_x + \sigma_y}{w} \right) \cos \theta = 0 \tag{20}
\]

\[
\sigma_r = \frac{\sigma_x + \sigma_y}{2} \tag{21}
\]

Sub. In Eq. (20)

\[
\frac{d \sigma_x}{d_x} + \frac{2 \mu \sigma_y}{h} \left( \frac{\sigma_x + \sigma_y}{w} \right) \cos \theta = 0 \tag{22}
\]

Sub. In Eq. (23) in Eq. (22)

\[
\frac{d \sigma_x}{d_x} + \frac{\mu \sigma_x \cos \theta}{w} + \frac{\sigma_y \left( \frac{2 \mu}{h} \frac{\mu \cos \theta}{w} \right)}{w} = 0 \tag{22}
\]

\[
\sigma = \frac{\sqrt{3}}{2} (\sigma_x - \sigma_y) \Rightarrow \sigma_y = \sigma_x - \frac{2 \sigma}{\sqrt{3}} \tag{23}
\]
\[
\frac{d \sigma_x}{dx} + \frac{\mu \sigma_y \cos \theta}{w} + (\mu \frac{2 \sigma_0}{\sqrt{3} h} + \frac{2 \mu \cos \theta}{w}) = 0
\]

\[
\frac{d \sigma_x}{dx} + \frac{\mu \sigma_x \cos \theta}{w} + \frac{2 \sigma_x}{h} + \frac{\sigma_x \mu \cos \theta}{w} - \frac{4 \mu \sigma_0}{\sqrt{3} h} \frac{2 \mu \cos \theta}{\sqrt{3} w}
\]

\[
\frac{d \sigma_x}{dx} + 2 \mu w_1 \left( \frac{1}{h} + \frac{\cos \theta}{w} \right) = \sigma \left( \frac{4 \mu}{\sqrt{3} h} + \frac{2 \mu \cos \theta}{\sqrt{3} w} \right)
\]

Consider \( \sigma \) is not function of \( x \), then:

\[
\frac{d \sigma_x}{dx} + \beta_1 \sigma_x - \beta_2 = 0
\]

Where the constants \( \beta_1 \) and \( \beta_2 \) are:

\[
\beta_1 = 2 \mu \left( \frac{1}{h} + \frac{\cos \theta}{w} \right)
\]

\[
\beta_2 = \sigma \left( \frac{4 \mu}{\sqrt{3} h} + \frac{2 \mu \cos \theta}{\sqrt{3} w} \right)
\]

Solution of this differential equation is:

\[
\sigma_x = \frac{\beta_2}{\beta_1} + M e^{-\beta_1 x}
\]

\[
M = \frac{\beta_2}{\beta_1} + \frac{\beta_1 e^{\beta_1 x}}{\beta_1}
\]

\[
\sigma_x = \frac{\beta_2}{\beta_1} e^{\beta_1 x}
\]

From Eq. (23)

\[
2 \bar{\sigma} \frac{\sigma_y}{\sqrt{3}} + \sigma_y = \frac{\beta_2}{\beta_1} \left( 1 - e^{\beta_1 (\ell - x)} \right)
\]

\[
\sigma_y = \frac{2 \bar{\sigma}}{\sqrt{3}} \left[ \frac{1}{h} + \frac{\cos \theta}{2 w} \right] \frac{\beta_2}{\beta_1} \left( 1 - e^{\beta_1 (\ell - x)} \right) \frac{2 \sigma_0}{\sqrt{3}} \frac{2 \mu \cos \theta}{\sqrt{3} w}
\]

\[
\sigma_y = \frac{2 \bar{\sigma}}{\sqrt{3}} \left[ \frac{1}{h} + \frac{\cos \theta}{2 w} \right] \frac{\beta_2}{\beta_1} \left( 1 - e^{\beta_1 (\ell - x)} \right) \frac{1}{h + \frac{\cos \theta}{w}}
\]

\[
\text{Load} = \frac{P}{P_{av}} \text{ w/N}
\]

\[
L_{\text{tooth}} = N \int_{\ell}^{0} \sigma_y wdx
\]
\[ N = \frac{2}{\sqrt{3}} w \int_{0}^{\ell} \left[ \beta_{1} e^{\beta_{1}(L-x)} - 1 \right] dx \]

\[ \frac{2N w_{0}}{\sqrt{3}} \left[ (y - 1) - \beta_{1} \int_{0}^{\ell} \frac{1}{\beta_{1}} e^{\beta_{1}(L-x)} dx \right] \]

\[ \frac{2N w_{0}}{\sqrt{3}} \left[ (y - 1) - \beta_{1} \left( \frac{1}{\beta_{1}} - e^{\beta_{1}(L-x)} \right) \right] \]

From Eq. (28)

\[ \frac{P_{av}}{\sigma} \left[ \frac{P_{av}}{\sigma} \right]_{\text{tooth}} = \frac{2}{\sqrt{3}} \left[ (y - 1) + \beta_{1} \left( 1 - e^{\beta_{1}} \right) \right] \]

According to the previous analysis with the change in variables \((C_{1}, C)\) to the new variables \((\beta, \beta_{1})\) we can find the axisymmetric portion:

\[ \frac{P_{av}}{\sigma} \left[ \frac{P_{av}}{\sigma} \right]_{\text{axi.}} = \frac{2}{\sqrt{3}} \left[ \frac{1}{\beta_{1}} \left( 1 - e^{\beta_{1}} \right) - 1 \right] e^{D_{1}D_{1}} \]

* For the constant volume: 

\[ \pi \frac{r_{2}^{2}}{2} h_{o} = \frac{r_{2}^{2}}{2} h + (Nw / h + N \left( \frac{\sin \theta}{\sin (90-\theta)} \right) h) \]

\[ \pi \frac{r_{2}^{2}}{2} h_{o} = \pi \frac{r_{2}^{2}}{2} h + \pi \frac{r_{2}}{h} \left( \frac{h_{o} - h}{h} \right) + N h / \left( \frac{\sin \theta}{\sin (90-\theta)} \right) \]

Where:

\[ w = \beta \frac{r_{2}^{2}}{2} \quad \beta = \frac{\pi}{N} \]

\[ \pi \frac{r_{2}^{2}}{2} (h_{o} - h) = \ell (\pi r_{2} h + Nh) \left( \frac{\sin \theta}{\sin (90-\theta)} \right) \]

\[ \ell = \frac{\pi \frac{r_{2}^{2}}{2} (h_{o} - h)}{\pi r_{2} h + Nh \left( \frac{\sin \theta}{\sin (90-\theta)} \right)} \]
Tooth behavior and state of Stresses

Fig. 1 the coordinate systems and the state of stress on the tooth of the gear.

Fig. 2 the three deformation regions forming one deformation unit

Fig. 3: Modification of the angle $\theta$ on the teeth.
COMPUTER PROGRAM RESULTS WHERE THE EFFECT OF ANGLE Θ IS NOT CONSIDERED
Analysis of Closed Die Cold Forging for Spur Gear Using Three Dimensional Slab Technique

FIG. #14
First Run

FIG. #15
First Run

Fig. #16
Second Run

Fig. #17
Second Run

Fig. #18
Third Run

Fig. #19
Third Run
1. Experimental Results and Discussion

The forming die set, punch and die, have been designed and manufactured at the workshop of the Mechanical Engineering Department, Kuwait University. Details of the die parts and the billet are hown in attached catalogue.

- For the experimental work:
  \[ D_i = \sqrt{D_o^2 \cdot h_o / h_i} \]
  \[ \varepsilon_{bar} = -\ln(h_i / h_o) = 2 \cdot \ln(D_i / D_o) \]
  \[ \sigma_{bar} = \sqrt{\frac{L_i \cdot \pi}{4}} \cdot D_i^2 \]
  Where:
  
  \( D_i \): Calculated diameter.
  \( D_o \): Initial diameter.
  \( L_i \): Applied load.
  \( h_i \): Final height
  \( h_o \): Initial height.
  \( \varepsilon_{bar} \): \( \varepsilon \)
  \( \sigma_{bar} \): \( \sigma \)

“PTOT” is \( \frac{P_{av}}{\sigma} \)

*Measurement of \( (D_h) \):

\[ h \cdot (L_1 + L_2) / 2 \cdot L \cdot N = (D_h) \cdot P_1 \cdot r_2^2 \]
\[ h \cdot (L_1 + L_2) / 2 \cdot L \cdot N = (h_o - h) \cdot P_1 \cdot r_2^2 \]

\( L_1 = 3 \text{ mm} \)
\( L_2 = 5 \text{ mm} \)
\( L = 5 \text{ mm} \)
\( h = 10 \text{ mm} \)
\[ 10 \cdot (3+5) / 2 \cdot 5 \cdot 12 = (h_o - 10) \cdot P_1 \cdot (16)^2 \]
Solving this equation you get:
\( h_o = 2.9841 \text{ mm} \)

CNC Works
Die part done by the CNC ET-Trak plus which is programming, operating & care manual. It acts like an advanced digital readout in manual machine operation, it acts like a CNC when programmed to do complex contouring, and it acts with the best qualities of each, when your job is best done by transitioning back and forth between manual and contouring CNC operations with the powerful DO ONE routines.

![Sample of gear used in test](image)

**Compression test**

*Al. Billet with hardness of 63 RB (Before heat treatment)*

* With different type of lubricant.
* There is small deflection in the plates:
  * In the small plate = 0.4 mm
  * In the big plate = 0.34 mm
* The hardness of the Aluminum material:
  * = 63 RB
* The specimen shape as follows:

![Al specimen](image)

\[ h_o = 13.56 \text{ mm} \]
\[ \text{av. Do} = 32.08 \text{ mm} \]

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<tr>
<th>Point #</th>
<th>Load (KN)</th>
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<th>D exp (mm)</th>
<th>Di (mm)</th>
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\[ h_f = 10.6 \text{ mm} \]
\[ D_f = 36.34 \text{ mm} \]
* There is small reduction in the height after reload.
**ANALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE**

![Fig.13 Specimen # 1](image)

**Specimen # 2**

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<th>Load (KN)</th>
<th>Displ. (mm)</th>
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<th>Di (mm)</th>
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* hf = 10.89 mm
  Df = 36.34 mm

* There is small reduction in the height after reload.
  (more barell)

![Fig.14 Specimen # 2](image)

**Specimen # 2**

**Specimen # 3**

Lubricant : Sheet of plastic av.

- ho = 13.62 mm
- Do = 32.102 mm
- hf = 10.25 mm
- Df = 35.66 mm
**Analysis of Closed Die Cold Forging for Spur Gear Using Three Dimensional Slab Technique**

<table>
<thead>
<tr>
<th>Point #</th>
<th>Load (Li) (KN)</th>
<th>Displ. (Di) (mm)</th>
<th>Epsilon-bar</th>
<th>Sigma-bar (Pa)</th>
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* There is small reduction in the height after reload.

**Fig.15 Specimen # 3**

- Forging process
- Al. Billet with hardness of 29 RB
- (After heat treatment)
- * With oil lubricant.

**Specimen # 1**

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<th>Point #</th>
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Lubricant: Oil

av. ho = 13.46 mm

Do = 31.912 mm

*Fig. 15 Specimen # 3* (without hole)
NALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE

Specimen # 2
(With 5 mm hole in the middle of the billet)
Lubricant : Oil
av. ho = 13.745 mm
Do = 32.09 mm
Lubricant : Oil

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<th>Di (mm)</th>
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Max. load = 520 KN

Specimen # 3
(With 10 mm hole in the middle of the billet)
Lubricant : Oil
av. ho = 13.5 mm
**Analysis of Closed Die Cold Forging for Spur Gear Using Three Dimensional Slab Technique**

*Do = 31.94 mm*

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Max. load = 506 KN

![Fig.18 Specimen # 3](image1.png)

![Fig.19 Forging process](image2.png)
II. DISCUSSION

- In the theoretical graphs the progressive increase of tooth length is plotted against the reduction in billet, tooth length is increase with the increase of reduction in billet height for all runs and values.
- The numerical values of the relative average punch pressure ($\text{Pay.}/\sigma$) are determined for 12 teethes with root diameter of 32 mm by using slab method. The results are obtained by the slab method shown in Figures 4 to 22. These results are obtained for three values of coefficient of friction t such as (.1, .2 and .3) and three values of number of teeth N such as (12, 20 and 30). It is clearly shown that the relative average punch pressure ($\text{Pay.}/\sigma$) is increasing in an exponential form as the reduction in height of the billet increases.
- As the reduction in height increase the load needed is increase. Lubricants affect the compression test, where the value of $\sigma$ is decrease when we add lubricant and we get red of barreling in the billet. The load decrease after heat treatment on the billet where the hardness of the material is decrease.
- As the effective strain increase the effective stress increase.
- Flow stress of the material was known by the compression test, with and without lubricant, before and after heat treatment.
- Finally a. comparison between the experimental and the theoretical shown from Fig.( 6) and Fig.(46). It is possible to see from the results shown that the slab method underestimate the values of ($\text{Pay.}/\sigma$) for small values of reduction in height while it gives nearly closed values values for high values of reduction in height, which is seen it is good agreement within approximately 36% of error which is not to high.

III. CONCLUSION

A kinematically admissible velocity field taking into account the sidewise spread as well as the bulging along thickness has been proposed for upset forging of arbitrarily shaped prismatic blocks. The flexibility of the proposed method has been demonstrated by analyzing upset forging of clover-shaped and rounded rectangular blocks. For different experimental conditions in lubrication and billet shape, there is a good agreement between the theoretical forging load and the experimental load for Aluminum. The theoretical prediction of the deformed configuration is to some extent in good agreement with the experimental measurement except for high friction. The velocity field proposed in the present investigation can be used for the prediction of forging load and deformation in upset forging of arbitrarily shaped prismatic blocks.

REFERENCES

NALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE


Appendix: A

Computer Program Setup

The following computer program has been prepared to solve the previous equations in order to calculate (pav./σ) which is denoted by PTOT throughout the program. Five computer runs have been performed according to the date accompanying each run.

REM " THIS PROGRAM IS TO CALCULATE (Pav. / SEGMA)
REM OF A GEAR WITH THE VARIETY OF H"
REM (pav./SEGMA) tot = (Pav./SEGMA) tooth + (Pav./SEGMA) axi"
REM SEGMA = SEGMA - BAR"

DIM Dh (500), PTOT (500), L (500)
GLS
PRINT " A COMPUTER PROGRAM"
PRINT " DONE BY : ALI AL-KHAMIS"
PRINT " "
PRINT " ENTER THE NO. OF TEETH N" ; N
PRINT " ENTER THE VALUE OF INNER RADIUS OF BILLET (r2) IN m"; r2
PRINT " ENTER COEFFICIENT OF FRICTION: ; MU"
PRINT " ENTER THE VALUE OF INITIAL HEIGHT (ho) IN m"; ho

Appendix: A

Computer Program Setup

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PRINT " DONE BY : ALI AL-KHAMIS"
PRINT " "
PRINT " ENTER THE NO. OF TEETH N" ; N
PRINT " ENTER THE VALUE OF INNER RADIUS OF BILLET (r2) IN m"; r2
PRINT " ENTER COEFFICIENT OF FRICTION: ; MU"
PRINT " ENTER THE VALUE OF INITIAL HEIGHT (ho) IN m"; ho
PRINT " THE VALUE OF N = " ; N
PRINT " THE VALUE OF r2 = " ; r2
PRINT " THE VALUE OF MU = " ; MU
PRINT " THE VALUE OF ho = " ; ho

Z = 0
PRINT " h PT P axi "
PRINT "-----------------------------
FOR h = 13.5 TO 10 STEP -.2
Z = Z + 1
W = (3.141592654 # / N) * Rw
A = (1 / w) + (1 / h)
B = (1 / h) + (1 / (2 * w))
C = B / A
Cl = 2 * MU * A
IZ) = r2 * ((ho / h) - 1)
I1 = 1 - EXP (c1 * L(Z))
PT2 = c / (c1 * L(Z))
PT3 = c - 1
    = 1.1547 * (PT3 + (PT2 * PT1))
D1 = (2 * MU * r2) / H
D2 = (2 * MU) / h
L = (r2 / D2) + (1 / D2^2)
L2 = -1 * EXP (-D2 * r2)
L3 = 1 / D2^2
    = (L2 * L1) + L3
PA1 = EXP (D1) * I
PA2 = EXP (D1) * I
PA2 = 1.1547 * c * (1 - EXP (c1 * L(z)))
PA3 = 2 / r2^2
Axi = PA3 * (PA2 - 1) * PA1
PTOT(Z) = ABS (Paxi + PT)
Dh (z) = ho - h
PRINT h; " PT; " ; Paxi
NEXT h
PRINT " PTOT D (h) L(h)"
PRINT "-------------------------------------"
QA = 1 TO Z
PRINT PTOT (QA), Dh (QA), L(QA)
NEXT QA
SCREEN 9
WINDOW (—15, —15) —(25, 25)
LINE (—15, —15) —(30, 30), 0, BF
LNE (—15, —15) —(30, 30), 12, B
LINE (—15, 0) —(30, 0), 2
LINE (0, —15) —(0, 30), 2
LOCATE 2, 13: PRINT “Pav./SEGMA-BAR”
LOCATE 17, 73 PRINT “Dh/ho”
LOCATE 16, 28 PRINT “(0,0)”
LOCATE 22, 18: PRINT “FIFTH RUN”"
LINE (5, 0) —(5, 1), 8
LINE (10, 0) —(10, 1), 8
LINE (15, 0) —(15, 1), 8
LINE (20, 0) —(20, 1), 8
LOCATE 15, 38 PRINT “ 005”
LOCATE 15, 48 PRINT “ 01”
LOCATE 15, 58 PRINT “ 015”
LOCATE 15, 68: PRINT “.02”
LINE (0, 5) —(1, 5), 3
LINE (0, 10) —(1, 10), 3
NALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE

LINE (0, 15)—(1, 15), 3
LINE (0, 20)—(1, 20), 3
LOCATE 10, 25 PRINT “2”
LOCATE 7, 25: PRINT “3”
LOCATE 13, 25 PRINT “3”
LOCATE 4, 25: PRINT “4”
FOR P = 1 TO Z
IF PTOT(P) > 999 THEN
   ( ) TO 1
ELSE
   LINE (Dh(P) / ho * 15, PTOT(P) * 5)—(Dh(P + 1) / ho * 15, PTOT(P + 1) * 5), 6
END IF
NEXT P
A$ = INPUT$(1)
CLS
LINE (—15, —15)—(105, 40), 0, BF
LINE (—15, —15)—(105, 40), 4, B
LINE (—10, 0)—(85, 0), 8
LINE (0, —10)—(0, 85), 8
LOCATE 2, 25 PRINT “ L/r2”
LOCATE 17, 73: PRINT “Dh/ho”
LOCATE 22, 18: PRINT “FIFTH RUN”
LINE (0, 5)—(1, 5), 3
LINE (0, 10)—(1, 10), 3
LINE (0, 15)—(1, 15), 3
LINE (0, 20)—(1, 20), 3
LOCATE 15, 38: PRINT “.005”
LOCATE 15, 48: PRINT “.01”
LOCATE 15, 58: PRINT “.015”
LOCATE 15, 68: PRINT “.02”
LOCATE 10, 25: PRINT “.032”
LOCATE 7, 25: PRINT “.048”
LOCATE 13, 25: PRINT “.016”
LOCATE 4, 25: PRINT “.064”

LINE (5, 0)—(5, 1), 8
LINE (10, 0)—(10, 1), 8
LINE (15, 0)—(15, 1), 8
LINE (20, 0)—(20, 1), 8
FOR R = 1 TO Z
   PTOT(R) > 9000 THEN
      ( ) TO 2
   ELSE
      LINE (Dh(R) / ho * 60, L(R) / r2 * 60)—(Dh(R + 1) / ho * 60, L(R + 1) / r2 * 60: 6
   END IF
NEXT R
A$ = INPUT$(1)
2A$ = INPUT$(1)
LOCATE 10, 32: PRINT “ THE END”
LOCATE 19, 25: PRINT “***** ALI AL-KHAMIS”
LOCATE 21, 25: PRINT “***** ALI AL-KHAMIS”
A$ = INPUT$(1)

Appendix: B

Computer program: With the effect of the angle θ on the teeth
REM “ THIS PROGRAM IS TO CALCULATE (Pav./SEGMA)
REM OF A GEAR WITH THE VARIETY OF h”
REM
NALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE

REM “(Pav./SEGMA)tot = (Pav./SEGMA)tooth + (Pav./SEGMA) axi ”
REM
REM “ SEGMA = SEGMA - BAR

DIM Dh(500), PTOT(500), L(500)
CLS
LPRINT “ A COMPUTER PROGRAM”
LPRINT “ DONE BY ALI AL-KHAMIS”
LPRINT “”
LPRINT “ = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
LPRINT “ ENTER THE NO. OF TEETH N”; N
INPUT “ENTER THE VALUE OF INNER RADIUS OF BILLET (r2) IN m”; r2
INPUT “ENTER COEFFICIENT OF FRICTION”; MU
INPUT “ENTER THE VALUE OF INTIAL HEIGHT (ho) IN m”; ho

LPRINT “THE VALUE OF N = “; N
LPRINT “THE VALUE OF r2 = “; r2
LPRINT “THE VALUE OF MU = “; MU
LPRINT “THE VALUE OF ho “; ho

z = 0
LPRINT “ h                                            PT                          Paxi “ “
PRINT “-------
-
-------------
FOR h = 13.5 TO 10 STEP -.2
Z = Z + 1
= (3.141592654 # / N) * r2
A = (.9799247 # / W) + (1 / h)
B = (1 / h) + (.9799247# / (2 * W))
C = B / A t.i = 2 * MU * A
C1  =  2  *  MU  *  A
F = 3.14159265# * r2   ^   2 * (ho — h)
M = 3.14159265# * r2 * h
K  = .2034523# * N * h
L(Z) = F / (M + X)
PT1 = 1 - EXP(c1 * L(Z))
PT2 = c / (c1 * L(Z))
PT3 = c - 1
PT = 1.1547 * (PT3 + (PT2 * PT1))
1 = (2 * MU * r2) / h
2 = (2 * MU) / h
L1 = (r2 / D2) + (1 / D2 ^ 2)
L2 = —1 * EXP(—D2 * r2)
L3= 1 / D2 ^ 2
1 = (L2 * L1) + L3
PA1 = EXP(Di) *
PA2 = 1.1547 * c * (i — EXP(c1 * L(Z)))
PA3 = 2 / r2 ^ 2
Paxi = PA3 * (PA2 - i) * PA1
PTOT(Z) = ABS(Paxi + PT)
Ph(Z) = ho - h
LPRINT h; “ ; PT ; “ ; Paxi
NEXT h
LPRINT “ PTOT” D(h) / ho 1 (h) / r2"
LPRINT “---------------------------------------------------------------”
FOR QA  = 1 TO Z
LPRINT PTOT (QA), Dh (QA) / ho, L(QA) / r2
NEXT QA
ANALYSIS OF CLOSED DIE COLD FORGING FOR SPUR GEAR USING THREE DIMENSIONAL SLAB TECHNIQUE