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Free Vibration of High Rise Buildings using Condensation of Matrices

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ABSTRACT

The complex configuration and free-form shapes of many DOFs lead to an increase in the size of the matrices. As a result, the dynamic analysis of high-rise buildings becomes more complex. These buildings need an extension of more time in the analysis process, and super-computers and costs significantly increased. Accordingly, the reduction of the size of the matrices is required. Therefore, condensation of matrices is one of the most efficient techniques to solve this problem. This research investigates a condensation technique to simplify free vibration dynamic analysis of high-rise buildings to reduce the big matrices resulting from many DOFs based on the Guyan-Irons reduction. The method suggests two approaches (Static and Dynamic) condensation methods. These methods reduce the size of the whole dimension of the structural matrices. A comparison between the static and the dynamic condensation methods was carried out, and obtained the natural frequency in both cases. The results revealed that dynamic condensation is more efficient in calculating the frequencies and mode shapes than static condensation since the dynamic condensation methods consider the effects of the inertia terms of the ignored DOFs, in contrast to static condensation. Because the inertia terms are related to the inverse of the dynamic stiffness matrix, it is impossible to obtain this matrix directly. The dynamic condensation for analyzed models gives a maximum deviation of ± 5:7% from Sap 2000. Therefore, dynamic condensation is applied to sizeable finite element models to compute the frequencies and the different mode shapes faster.

KEYWORDS: Static condensation, Dynamic condensation, High-Rise Buildings, Free Vibrations.

الاهتزازات الحرة للمنشآت العالية باستخدام تكثيف المصفوفات

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الملخص

أصبح الاهتمام بالمباني الشاهقة من أهم أولويات الدول المتقدمة كمتطلبات النمو السكاني، تسعى كل دولة لتكون صاحبة أطول ناطحة سحاب. لذلك، تحالف الابتكار المعماري والإنشائي لتصميم ونمذجة وتحليل الهياكل المعقدة. اختلفت الأنظمة الإنشائية لتحمل الأحمال الجانبية. فبزيادة عدد الطوابق والخلجان وأنظمة البناء المختلفة، واختلاف مواد البناء تزيد عدد درجات الحرية، وبالتالي أصبحت المصفوفات كبيرة ومعقدة، واستغرق حلها مزيدًا من الوقت والجهد. حالياً الاهتمام بإيجاد حل لتقليل أبعاد المصفوفات لان التحليل الديناميكي للمباني الشاهقة صار أكثر تعقيدًا بسبب التكوين المعقد وعدد درجات الحرية الكبير ، أحد الحلول هو تكثيف المصفوفات. لتصبح أسهل وأكثر اقتصادا في حل البرمجيات وأجهزة الكمبيوتر.

الهدف الرئيسي للبحث: تقليل أبعاد المصفوفات بتقنية التكثيف لتبسيط التحليل الديناميكي للاهتزازات الحرة للمباني الشاهقة لتقليل المصفوفات الكبيرة الناتجة عن العديد من DOF على أساس تقليل Guyan-Irons. باقتراح طريقتان (ثابت وديناميكي) للتكثيف. تم إجراء مقارنة بين طريقتي التكثيف والحصول

على التردد الطبيعي في كلتا الحالتين ببرنامجFORTRAN . أيضا دراسة الفرق بين الاختيار الدقيق لدرجات الحرية الرئيسة والمحذوفة ومقارنة النتائج. أظهرت النتائج أن التكثيف الديناميكي أكثر كفاءة في حساب الترددات وأشكال الأنماط من التكثيف الساكن حيث أن التكثيف الديناميكي تأخذ في الاعتبار تأثيرات شروط القصور الذاتي في DOFs المتجاهلة ، على عكس التكثيف الثابت. نظرًا لأن مصطلحات القصور الذاتي مرتبطة بعكس مصفوفة الصلابة الديناميكية ، فمن المستحيل الحصول على هذه المصفوفة مباشرةً. يعطي التكثيف الديناميكي للنماذج التي تم تحليلها انحرافًا أقصى قدره ± 5: 7٪ عن Sap 2000. لذلك ، يتم تطبيق التكثيف الديناميكي على نماذج العناصر المحدودة الكبيرة لحساب الترددات وأشكال الوضع المختلفة أسرع. استخدام مؤخرًا في ارتباط نماذج تحليل الاختبار والتحكم في الاهتزاز والتحسين الديناميكي الهيكلي وتحليل الاستجابة الديناميكية. تم إجراء تحليل حر لايجاد التردد الطبيعي - الاهتزاز الحر له دورًا مهمًا في التصميم - خاصة للحالة الأساسية لأنه عنصر مهيمن في اهتزازات الرياح والزلازل، حيث يتصرف الهيكل بمرونة.

الكلمات المفتاحية : التكثيف الاستاتيكي، التكثيف الديناميكي، المباني الشاهقة، التحليل الحر.

1. INTRODUCTION

Model reduction techniques have been widely used in structural dynamics, analysis, reanalysis, structural dynamic optimization, eigenvalue problem, model update, and damage detection. Condensation is not only limited to designing high-rise buildings to reduce FEM but also helps in different fields of civil engineering, such as Structural Health Monitoring SHM, Experimental Model Analysis, and FEM Experimental Correlation. Detection of damage to installations is essential. Timely detection of damage ensures safety and achieves economic considerations. The structure contains many DOFs, which are difficult to measure due to high experiment costs. Therefore, model reduction techniques are applied to reduce the DOFs [1 - 6].

The construction of high-rise buildings worldwide has become imperative. Some high-rise buildings are urban landmarks of a city. Currently, every country is striving to be the owner of the tallest skyscraper in the world. Urban development through quickened expansion of large cities requires structural design, increasingly complex calculations, and modeling procedures. Structural systems differ (Rigid frames, Braced, shear-walled frames, Outrigger, Framed-tube, Braced-tube systems, etc.) to withstand lateral loads that may cause damage to high-rise buildings. Increasing the number of stories and bays, the different construction systems and their diversity, and the different construction materials increase the DOFs, and thus the matrices became large and complex, taking more time and effort to solve. So, the dynamic characteristics of the structures become more complex, and the corresponding computational time and costs increase dramatically. This research solves the dynamic analysis as undamped free analysis. The free analysis is one of the modal analysis types (the essential type of dynamic analysis) to determine the structure's natural frequencies and mode shapes. It solves the structural system's eigenvalue problem to find its dynamic properties.

Free vibration analysis plays an essential part in the structural design of buildings, especially for the fundamental mode, because this mode is a dominant component in tall buildings' wind- and earthquake-induced vibrations. While the structure behaves elastically, the maximum response acceleration will depend on the structural natural period of vibration and the magnitude of the damping present. In dynamic F.E. analysis, the eigenvalue problem results from a complex structure, so it is eligible to minimize the number of DOFs. Recently, structural engineers have been very interested in finding a solution to reduce the dimensions of these matrices. One of these solutions is the condensation of matrices. Previous studies are interested in developing them to solve dynamic problems. To become more accessible and more economical in the solution of software and computers. Reducing DOFs reduce the matrices by ignoring the unimportant and keeping only the essential DOFs, known as the masters. So, we have a smaller model than the large model, the computational efforts and time analysis decreased, and this method has recently taken many developments and modifications to be suitable for dynamic problems. There are two methods of reduction by condensation; the easier and more popular one is the Static Condensation Method which is appropriate in a static problem when ignoring the mass term completely for un-damped structures, but this method gets approximate. It may produce many things that could be improved when applied to dynamic problems. So, to improve this method to solve dynamic problems and consider the dynamic effect on the structure, this method gives practically exact results. This method is the Dynamic Condensation Method to be more appropriate for dynamic problems.

Dynamic Condensation Method classification to 1-single-step method: The condensation matrix is generated first and then used to calculate the reduced stiffness and mass matrices, 2-two-step method: Used single-step method as an approximate estimation of the reduced model, finally make adjustments to compensate for the ignored effects, and 3-multi-step method: An iterative method in which modified the condensation matrix in successive iterations. The Guyan–Iron method is the most known static reduction method, which reduces the inner nodes of F.E. models statically to boundary nodes. Guyan [7] put the basics necessary to reduce matrices' size and reduce non-diagonal mass matrices for natural mode analysis. Irons [8] presented a technique for reducing the elements needed to compute eigenvalues for full matrices, which is used in most methods to condense matrices subsequently. Guyan is exact for static problems, but it does not succeed in reducing a system without damaging the system's dynamical properties; its accuracy strongly depends on the choice and number of the boundary nodes describing the reduced system [36]. A new efficient method of dynamic condensation without approximation presented to solve the frequency result-dependent eigenvalue by Sturm sequence by some iterations; accuracy does not depend on selecting masters [9].

Paz [11] presented a technique for reduction based on the one-step scheme considered an extension of Guyan and applied it to the dynamic matrix, then solved the reduced eigenproblem; this method does not require matrix inversion or series expansion. When selecting the masters accurately, Guyan applies correctly, and the selection method must allow the limits of Guyan to be defined while keeping a minimum of a master [16]. Tried to get Dynamic DOFs using diagonal terms of the system's essential mass and stiffness matrices; the lower frequencies are not missed and get higher accuracy [12]. Presented a simplified dynamic condensation method (SDC) without modification [13]. O'Callahan proposed a new, improved reduced system (IRS) based on the two-step, using Guyan to obtain an approximate estimation of the reduced system matrices, then make some adjustments to compensate for the inertia effect, considering the first-order approximation terms in the transformation formula of the slave DOFs [15]. A new derivation technique of IRS presented, which must respond with a fundamental limitation in choosing neglected coordinates [17]. Blair and Camino apply IRS by proposing two modifications. An iterative method proposed with two assumptions [27]. IRS used to produce an iterative algorithm [21]. The iteration's convergence improved with three advantages: 1st, convergence is faster, 2nd more computationally efficient because it is not necessary to determine the stiffness and mass matrices every iteration and 3rd the convergence can be proved more efficient than subspace iteration for the first time [25, 26]. An iterative dynamic condensation technique derived by comparing it with all iterative schemes in the past [25]. When dynamic condensation is independent of reduction eigenvalues, it's optional to calculate every iteration, which makes the iterative very active, so it confirms the dynamic condensation matrix is a system feature. It doesn't influence by external forces [29]. An iterative dynamic condensation method uses the kept and reduced DOFs associated with a condensation matrix to get a condensed eigenvalue [18]. Condense the stiffness matrix exactly as obtained by Guyan, but the two mass matrices are different and can't accurately preserve higher modes of interest in the condensed model; this method can retain lower and higher modes with high accuracy [19]. An iterative method combining dynamic condensation method, modified subspace iteration, and modal reduction, the main advantage is that several eigenpairs can be with solution accuracy, incorrectly DOFs have little effect on overall matrices [23]. The measured frequencies can be used as an approximate solution in dynamic condensation to find an exact real eigenvalue [24]. In an iterative Dynamic Condensation technique, the accuracy of the reduced order system obtained is much higher than which of Guyan; also, the reduced system is updated repeatedly until the desired one is received [31]. An un-symmetric and damped reduction using an iterative dynamic condensation technique; uses orthonormalized complex eigenvectors of unsymmetrical systems to obtain the eigensolution of the reduced model by the Lanczos algorithm [33]. A new iterating dynamic condensation suggested that retains all the inertia terms related to the removed DOFs, which results in a reduced mass matrix [34]. Presented three condensation methods of non-classically damped systems [35, 36]. Improve dynamic condensation method by IIRS to modify the iterative transformation matrix and achieve faster convergence; linked with the subspace iteration method (SIM) proved that this method obtains the lowest eigensolutions more efficiently and accurately than IIRS [37]. Used an alternative dynamic condensation method for active vibration; compared with Guyan with many advantages, especially at high-frequency range [40]. Improved an iterated reduced system IIRS based on Friswell for un-damped and non-classically damped [42]. Static analysis with repeated patterns for the problem of inverting a matrix, a closed-form solution is applied. [44]. A developed multi-level condensation method to improve the efficiency of traditional matrix condensation by static analysis with repeated patterns [45]. Proposed a simplified dynamic condensation method to calculate the responses as a new damage detection method that uses only one mode shape and its corresponding eigenvalue [46]. Reduced the model to half size by using a simple method that uses the linear solver to get the much smaller matrix, using Cholesky to reduce the stiffness matrix [49]. The global mass and stiffness matrices are automatically divided into many small submatrices by deleting relatively large FE, for which the IRS failed, and worked to solve FE models with more than millions DOFs [50]. A non-iterative method, 'Maclaurin Expansion of the frequency response function in Laplace Domain' (MELD), was applied for the dynamic reduction of non-classically damped [52]. Extended static condensation method; applied to dynamic analysis, especially in linear elastic dynamics [53]. Develop a dynamic condensation method by selecting a few DOFs as a master, calculating responses, and deriving the response sensitivities [54]. Comparing the accuracy of different reduction techniques between Guyan, IRS, IIRS, and SEREP, comparison of high to low frequency. The results prove: (1) The highest errors are from Guyan, followed by IRS and IIRS, with underestimated errors. (2) Errors are increased in Guyan, IRS, and IIRS at higher frequencies, but errors in IIRS are much less. (3) SEREP provided accurate results for all frequency ranges. [1]. By SEREPa, detected the damage using expanded mode shapes based on SEREP [2]. Solve damage detection problems as a new development scheme [3]. SEREP is used as a mode shape expansion method to solve the problem of determining damage, which requires more DOFs to be measured [4]. The two-step methods use the POD and RBF methods to reduce models and computational time [5]. Traditional optimization techniques, such as particle swarm optimization, simulated annealing SA, and genetic algorithm, are used continuously to detect damages; SA implementation includes damage detection, FE model updating, sensor optimization, and system identification [6].

The main object of this paper is to examine the effect of Static and Dynamic condensation on the dynamic analysis of undamped free vibration of high-rise buildings. Various buildings with different DOFs, such as (shear buildings and frames in 3D) are proposed using condensation techniques based on the Guyan-Irons reduction. The results are compared before and after condensation, whether (static or dynamic) using the FORTRAN program, and the FORTRAN program results were verified with the results of the SAP 2000 program before condensation.

2. METHODOLOGY

Model reduction techniques have been widely used in structural dynamics, reanalysis, structural dynamic optimization, eigenvalue problem, model update, and damage detection. The dynamic equation of motion is written as a set of linear second-order differential equations:

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = \{F(t)\} \tag{1}$$

The reduced dynamic equation of motion is written as:

$$[M_R]\{\ddot{Z}(t)\} + [C_R]\{\dot{Z}(t)\} + [K_R]\{Z(t)\} = \{F_R(t)\} \tag{2}$$

Where: M_R, C_R and K_R are the mass, damping, and stiffness matrices, respectively, of the reduced order model, and F_R is the equivalent force vector on the reduced model. They are defined as:

$$[M_R] = [T]^T\,[M\,]\,[T], \quad [C_R] = [T]^T\,[C\,]\,[T], \quad [K_R] = [T]^T\,[K\,]\,[T]\ \&\quad [F_R] = [T]^T\,[F\,]\,[T]$$

In the reduced model, the dynamic characteristics of a whole model can be kept within a frequency range. This model is helpful in further dynamic analyses, especially at repeated calculation.

1.1. Static Condensation Method

A- Static Condensation Method on the Static Problem

When using static analysis, it will become necessary to reduce the stiffness matrix $[K]$ only. It's by using Guyan [7] and Irons [8] to cancel ignored DOFs by separating into slaves and masters; the relation between them is by obtaining static relation between them to reduce stiffness matrix [K].

The stiffness Equation for structure: $$\begin{bmatrix} [Kss] & \cdots & [Ksp] \\ \vdots & \ddots & \vdots \\ [Kps] & \cdots & [Kpp] \end{bmatrix} \begin{bmatrix} \{u_0\} \\ \vdots \\ \{u_p\} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \vdots \\ \{F_p\} \end{bmatrix} \tag{3}$$

Let: $\begin{bmatrix} [Kss] & [Ksp] \\ [Kps] & [Kpp] \end{bmatrix} \begin{bmatrix} \{u_s\} \\ \{u_p\} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \{F_p\} \end{bmatrix}$ (4)

Where $\{u_s\}$ is the displacement vector corresponding to the DOFs to be reduced

and $\{u_p\}$ is the vector corresponding to remaining independent DOFs.

1st assume external forces= zero at secondary DOFs.

By multiplication in Eqn. (1) expands into (5) & (6).

$[Kss]\,\{u_s\} + [Ksp]\,\{u_p\} = \{0\}$ (5)

$[Kps]\,\{u_s\} + [Kpp]\,\{u_p\} = \{F_p\}$ (6)

$\{us\} = [\ \bar{T}\ ]\,\{u_p\}$ (7)

Where, $[\ \bar{T}\ ]$ is the transformation matrix,

$[\ \bar{T}\ ] = -[Kss]^{-1}[Ksp]$ (8)

$[\ \bar{K}\ ]\ \{u_p\} = \{F_p\}$ (9)

$[\ \bar{K}\ ] = [Kpp] - [Kps]\ [Kss]^{-1}\ [Kps]$ (10)

When, $\{u\} = [\,T\ ]\,\{u_p\}$ (11)

$\{u\} = \begin{bmatrix} \{u_s\} \\ \{u_p\} \end{bmatrix}$ & $[\,T\ ] = \begin{bmatrix} [\,T\,] \\ [\,\mathrm{I}\,] \end{bmatrix}$ (12)

Substituted in Eqns. (11) and (3) into. (4) and pre-multiplying by the transpose of [T] results in:

$[T]^T\ [K\ ]\ [T\ ]\{u_p\} = [\,[T]^T\ \ [\,\mathrm{I}\,]\,] \begin{bmatrix} \{0\} \\ \{F_p\} \end{bmatrix} \longrightarrow [T]^T\ [K\ ]\ [T\ ]\{u_p\} = \{F_p\}$

$[\ \bar{K}\ ] = [T]^T\ [K\ ]\ [T\ ]$ (13)

Solve, (1), (12) into (4) by (Gauss-Jordan elimination), stiffness equation has been reduced to:

$\begin{bmatrix} [\,\mathrm{I}\,] & -[\ \bar{T}\ ] \\ [0] & [\ \bar{K}\ ] \end{bmatrix} \begin{bmatrix} \{u_s\} \\ \{u_p\} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \{F_p\} \end{bmatrix}$ (14)

Gauss-Jordan elimination produces the transformation matrix $[\ \bar{T}\ ]$ and the reduced stiffness matrix$[\ \bar{K}\ ]$. So, to get $[\ \bar{K}\ ]$ substituted into Eqn. (13) directly.

Guyan ignores the dynamic effect, so this is exact for static problems only. However, it has been widely used in many static and dynamic problems. So, Guyan is the initial approximation of exact dynamic condensation. Based on Guyan features, its accuracy may improve besides partial and full inclusion of inertia effects: a- The best selection of the masters. b- Increase the number of masters.

Frequencies are normally satisfactory in the range of [0,0.3fs] (fs: smallest eigenfrequency).

B- Static Condensation Method on the Dynamic Problem

When using the static condensation for dynamic analysis it is necessary to reduce the mass $[M]$and damping $[C]$matrices such as reduced stiffness matrix$[K]$.

It is assumed that the same static relationship between secondary and primary DOFs remains effective in dynamic problems; transformation based on static condensation to reduce the stiffness matrix is also used in reducing mass and damping matrices; this method is not exact and introduces errors in results in dynamic problems. The errors depend on the relative number of DOFs reduced and on the specific selection of these, the reduced mass matrix is given by:

$[\ \bar{M}\ ] = \ [T]^T\ [M]\ [T\ ]$ (15), and $[\ \bar{C}\ ] = \ [T]^T\ [C\ ]\ [T\ ]$ (16)

Where, $[\ \bar{\mathrm{M}}\ ]$, $[\ \bar{C}\ ]$ is the reduced mass matrix and the reduced damped matrix respectively.

Guyan was able to get more accuracy; this method is based on the assumption of displacement and shifting reference frequency for all mode shapes, also; it doesn't consider the dynamic forces. The reduction of mass and damping matrices can be justified by potential elastic energy V and kinetic energy KE as:

$\mathrm{V}\ = \frac{1}{2}\ \{\ u\ \}^T[\mathrm{K}]\ \{\mathrm{u}\}$ (17) $\mathrm{KE} = \frac{1}{2}\ \{\ \dot{u}\ \}^T[\mathrm{M}]\ \{\dot{u}\}$ (18)

Virtual work δW_d by damping forces F_d = [C] $\{\dot{u}\}$ corresponding to virtual displacement $\{\delta_{\mathrm{u}}\}$as:

$$\delta W_d = \{\ d_u\ \}^T[\mathrm{C}]\ \{\dot{u}\} \tag{19}$$

By the transformation Eq. (11) the results are

$$\mathrm{V}\ = \frac{1}{2}\ \{\ \mathrm{u_p}\ \}^{\mathrm{T}}\ [\ \bar{K}\ ]\ [\mathrm{T}\ ]\{\mathrm{u_p}\} \tag{20}$$

$$\mathrm{KE} = \frac{1}{2}\ \{\ \dot{\mathrm{u}}_{\mathrm{p}}\}^{\mathrm{T}}\ [\ \bar{M}\ ]\ [\mathrm{T}\ ]\{\ \dot{\mathrm{u}}_{\mathrm{p}}\} \tag{21}$$

$$\delta W_d = \{\delta \mathrm{u_p}\ \}^T[\ \bar{C}\ ]\ [\mathrm{T}\ ]\{\ \dot{\mathrm{u}}_{\mathrm{p}}\} \tag{22}$$

The Eqns. (20), (21), and (22) potential energy, kinetic energy, and virtual work of damping forces in terms of $\{\mathrm{u_p}\}$. $[\ \bar{K}]$, $[\ \bar{M}\ ]$ and $[\ \bar{C}\ ]$ stiffness, mass, and damping matrices corresponding to primary DOFs $\{\mathrm{u_p}\}$, result in the same potential energy, kinetic energy, and virtual work calculated with all original coordinates. So, we can substitute in (13), (15), and (16) to get 3 reduced matrices.

1.2. Dynamic Condensation Method

In contrast to static, dynamic condensation consider the effects of inertia of ignored DOFs. Because the inertia is related to the inverse of the dynamic stiffness matrix, they cannot be obtained directly.

A- <u>The Simplified Dynamic Condensation Method (without Modified)</u>

At 1st take an approximate value or set it equal= 0 for the first eigenvalue ω_1^2.

The dynamic condensation on dynamic matrix: $[D_1] = \ [K] - \ \omega_1^2\ [M\ \ ]$, put ω_1^2=0, then solving reduced eigenproblem to find, ω_1^2 and ω_2^2. Solve with one practically exact eigenvalue and an approximate value for next order eigenvalue calculated at each step. This method doesn't require matrix inversion or series expansion. So, consider eigenvalues problem of a discrete system for which it is desired to reduce secondary DOFs $\{\mathrm{u_0}\}$ and retain primary DOFs $\{\mathrm{u_p}\}$.

The equations of free motion may be written in partitioned matrix form:

$$\begin{bmatrix}[Mss] & [Msp]\\ [Mps] & [Mpp]\end{bmatrix}\begin{bmatrix}\{\ \ddot{u}_s\}\\ \{\ddot{u}_p\}\end{bmatrix} + \begin{bmatrix}[Kss] & [Ksp]\\ [Kps] & [Kpp]\end{bmatrix}\begin{bmatrix}\{u_s\}\\ \{u_p\}\end{bmatrix} = \begin{bmatrix}\{0\}\\ \{0\}\end{bmatrix} \tag{23}$$

Put $\{u\} = \{U\}\sin\omega t$ $\longrightarrow$ $\begin{bmatrix}[Kss] - \omega_1^2[Mss] & [Ksp] - \omega_1^2[Msp]\\ [Kps] - \omega_1^2[Mps] & [Kpp] - \omega_1^2[Mpp]\end{bmatrix}\begin{bmatrix}\{u_s\}\\ \{u_p\}\end{bmatrix} = \begin{bmatrix}\{0\}\\ \{0\}\end{bmatrix}$ (24)

From Guess Jordan: $\begin{bmatrix}[I] & -[\ \bar{T}_i\ ]\\ [0] & [D_i]\end{bmatrix}\begin{bmatrix}\{U_s\}\\ \{U_p\}\end{bmatrix} = \begin{bmatrix}\{0\}\\ \{0\}\end{bmatrix}$ (25)

$\{U_s\} = [\ \ \bar{T}_i\ ]\ \{U_p\}$ (26) $\{\ U\ \}_i = [\mathrm{T}]\ \{U_p\}$ (27)

$$[\ T_i\ \ ] = \begin{bmatrix}[\ \ \bar{T}\ ]\\ [\ \ \bar{I}\ \ ]\end{bmatrix}\ \ \&\ \ \ \{U_i\} = \begin{bmatrix}\{U_s\}\\ \{U_p\}\end{bmatrix} \tag{28}$$

To reduce the Mass matrix: $[\ \ \bar{M}_i\ \ ] = \ \ [\ T_i]^T\ [M]\ [T_i]$ (29)

And to get stiffness matrix: $[\ \ \bar{K}_i\ ] = \ [\ \ \bar{D}_i\ ] + \ \ \omega_i^2\ [\ \ \bar{M}_i\ ]$ (30)

The solution of reduced Eigen problem: $[\ \ \bar{K}_i\ ] - \ \ \omega_i^2\ [\ \ \bar{M}_i\ ]\ |\ \{u_p\} = \{0\}$ (31)

Then get eigenvalues, corresponding eigenvectors for any mode, improved eigenvalues, corresponding eigenvector, and an approximation for next order eigenvalue ω_{i+1}^2.

The current study used the FORTRAN to apply condensation methods to different models. The program is first validated with the cases generated by Paz [11]. The results show good agreement

Selection of Master the Degrees of Freedom

The total DOFs of a whole model must divide into masters and slaves. Which and how many DOFs kept as masters are the base of selection? There are rules and conditions for choosing some masters because selection error will result in a significant error when applying condensation. Accuracy, completeness, symmetry, and practicality are substantial requirements in the selection. Thus, the reduced model accuracy is an important consideration when selecting masters because choice should make the reduced model as accurate as possible. With the ratio increase, the dynamic characteristics computed from the reduced model approach those of the whole model steadily. However, a large ratio will lead to expensive computational effort.

A- **Selection of the Masters of The Guyan Condensation**

The valid eigenvalue range of Guyan is (0, ω_c^2) which ω_c^2 is the lowest eigenvalue of the slave model. The approximate error of eigenvalues is inversely proportional to eigenvalue ω_c^2. If selected masters contain the main kinetic energy of each mode, dynamic characteristics between full and reduced models can be almost the same. Selection criteria for masters to ensure accuracy. Specially:

- For uniform material, relative displacement determines node kinetic energy of modes, and degrees of the node where maximum relative displacement occurs should be masters.
- At centralized mass regions, the equivalent mass may be the main influencing factor in kinetic energy, so DOFs at centralized mass locations should be masters.

B- **Physical-Type Condensation, The** masters should:

- keep the greatest possible strain energy information.
- participate in the whole sum of energies relating to inertial kinetic energy and external forces.

1-Levy (1971):
(a) Select the DOFs that have the largest entries in the mass matrix
(b) Select the DOFs with the largest movements in the modes of interest.

2- Ramsden and Stocker (1969):
Selected the masters associated with the larger concentrations of mass and flexible reasonably relative to other mass concentrations and fixed constraints.

3- Downs (1980): The masters should always be displacements rather than rotations.

3. Case Study

Example 1

For the shown a uniform 4-story shear building for all stories (m= 1 Ib. $\sec^2$/in & k= 327.35 Ib/in). The Eigenvalue ω_i^2 results were calculated using the FORTRAN program before and after the static and dynamic condensation. Then the results were compared, as shown in Table. (1) and chart in Fig. (2), between the natural frequencies before and after static and dynamic condensation by reducing u_1 & u_3:

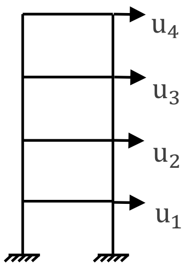


Fig.**1** 4-story Shear building Example [36].

$$[K] = 327.35\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad \& \ [M] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

TABLE 1. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for Shear building Example (4DOFs).

After Static and Dynamic Condensation of Coordinate $\mathbf{u_1}$ & $\mathbf{u_3}$:

	Before Cond.	Static Cond.	Error%	Dynamic Cond.	Error%
ω_1^2	**39.483**	**40.386**	2.2359	**40.386**	0.0229
ω_2^2	**327.350**	**365.98**	10.555	**329.137**	5.42935
ω_3^2	**768.388**				
ω_4^2	**1156.229**				

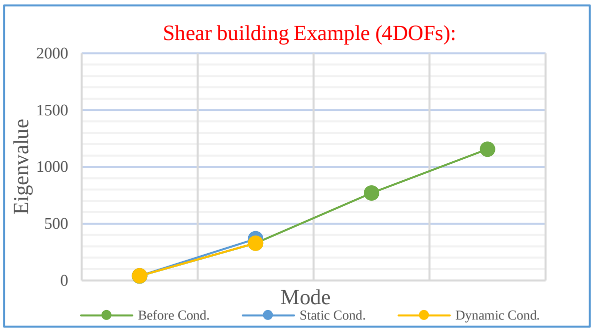


Fig.**2** Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for Shear building (4DOFs).

Example 2

The natural frequencies of a 20-story shear building with a height of 4 m for the story floor and 3 m for the repeated stories were calculated as shown in the Fig. (3) according to the moments of inertia of reinforced concrete column (25x25 cm) =.00032552 m^4 as a rectangular section with the uniform weight for each story= 4t. The natural frequency results were calculated using the FORTRAN program before and after the static and dynamic condensation. Then the results were compared, as shown in the following Table. (2) and Fig. (4), between the natural frequencies before and after the static and the dynamic condensation by eliminating 5 & 10 DOFs. For comparison between static and dynamic condensation, we find that dynamic condensation is more accurate and closer to the results without condensation. Also, when removing more degrees of freedom, the accuracy of the results decreases, as shown in Fig. (4).

TABLE 2. Comparison between FORTRAN Solution Before Cond., After Static & Dynamic Condensations after reduce 5 & 10 DOFs for 20-Story Shear building for W=4t.

Mode	Freq. Cps.	Condensation by eliminating 5 DOFs				Condensation by eliminating 10 DOFs			
		Static Cond.	*Error %*	*Dynamic Cond.*	*Error %*	*Static Cond.*	*Error %*	*Dynamic Cond.*	*Error%*
1	**.3046**	**.304**	0.197	**.3046**	0.0	**.3040**	0.197	**.3043**	0.0985
2	**.9154**	**.9185**	0.338	**.9159**	0.054	**.9261**	1.1554	**.9129**	0.2738
3	**1.5291**	**1.546**	1.093	**1.525**	0.2689	**1.570**	2.6051	**1.525**	0.2688
4	**2.1433**	**2.219**	3.411	**2.138**	0.2478	**2.203**	2.7099	**2.137**	0.2948
5	**2.7533**	**2.872**	4.133	**2.746**	0.2658	**2.958**	6.9202	**2.745**	0.3023
6	**3.3534**	**3.484**	3.748	**3.345**	0.2511	**3.844**	12.763	**3.345**	0.2511
7	**3.9384**	**4.370**	9.876	**3.933**	0.1373	**4.485**	12.187	**3.956**	0.4449
8	**4.5031**	**4.907**	8.2311	**4.514**	0.2414	**4.988**	9.7213	**4.535**	0.7034
9	**5.0434**	**5.353**	5.7836	**5.029**	0.2863	**5.686**	11.301	**5.135**	1.7838
10	**5.5550**	**6.445**	13.809	**5.539**	0.2889	**7.261**	23.495	**5.908**	5.9749
11	**6.0344**	**6.732**	10.362	**6.025**	0.156	-	-	-	-
12	**6.4783**	**6.962**	6.9477	**6.489**	0.1649	-	-	-	-
13	**6.8836**	**7.924**	13.129	**6.912**	0.418	-	-	-	-
14	**7.2477**	**8.002**	9.4263	**7.232**	0.217	-	-	-	-
15	**7.5681**	**8.064**	6.1495	**7.625**	0.752	-	-	-	-
16	**7.8428**	-	-	-	-	-	-	-	-
17	**8.0701**	-	-	-	-	-	-	-	-
18	**8.2483**	-	-	-	-	-	-	-	-
19	**8.3764**	-	-	-	-	-	-	-	-
20	**8.4537**	-	-	-	-	-	-	-	-

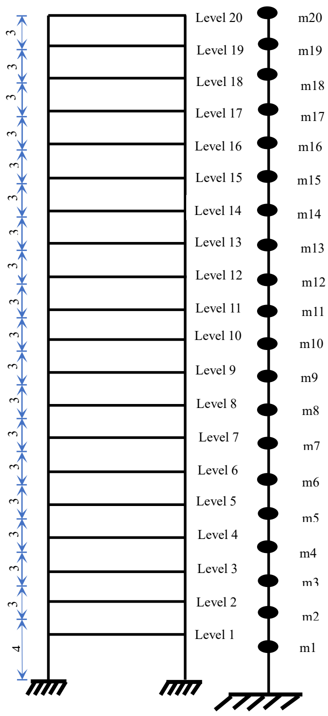


Fig.**3** The natural frequencies (Hz) of a 20-story shear building before and after static and dynamic condensation after reduce 5 & 10 DOFs.

Example 3

The building configuration adopted is simple and regular, as shown in Fig. (5). A 5-story steel frame building with reinforced concrete slabs and brick walls is used as a simulation model as a case study. The steel frames are arranged in X and Z directions with a square perimeter of 7m in the X-direction and 7m in the Z-direction. That is a square area (7x7) m^2.

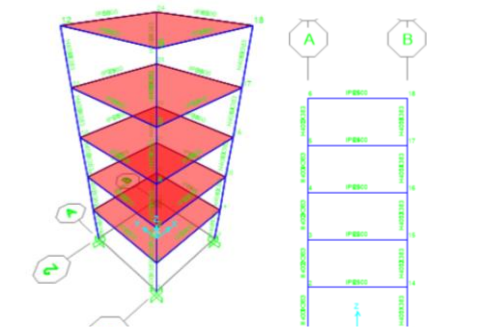
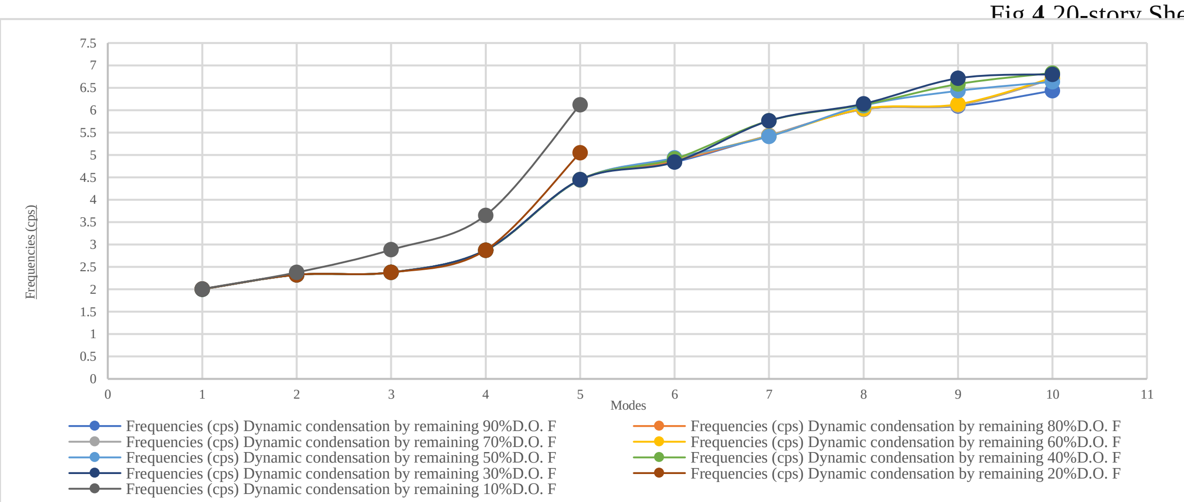


Fig.**6** A 5-story steel frame building.

The height of the story was H= 3 m. A grade of steel S355 was used with Fy =355MPa and Fu=480 Mpa, where the beam sectors were selected with sectors IPE (500) and the frame columns were selected with sectors H (400x383). The reinforced concrete of the solid slab is with standard concrete grade C30/37, and it is a square slab with a cover weight = 0.15t/m^2. The building mass is from the dead load, and the percentage is 25% from the live load, as the SAP2000 is also solved by (Model Mode). The natural frequency (cps) results were calculated using the FORTRAN program by Prof. Dr. Mohamed Naguib [22] before and after the static and the dynamic condensation 90, 80, 70, 60, 50, 40, 30, 20, and 10% respectively, this is by

Fig 4 20-story Shear building

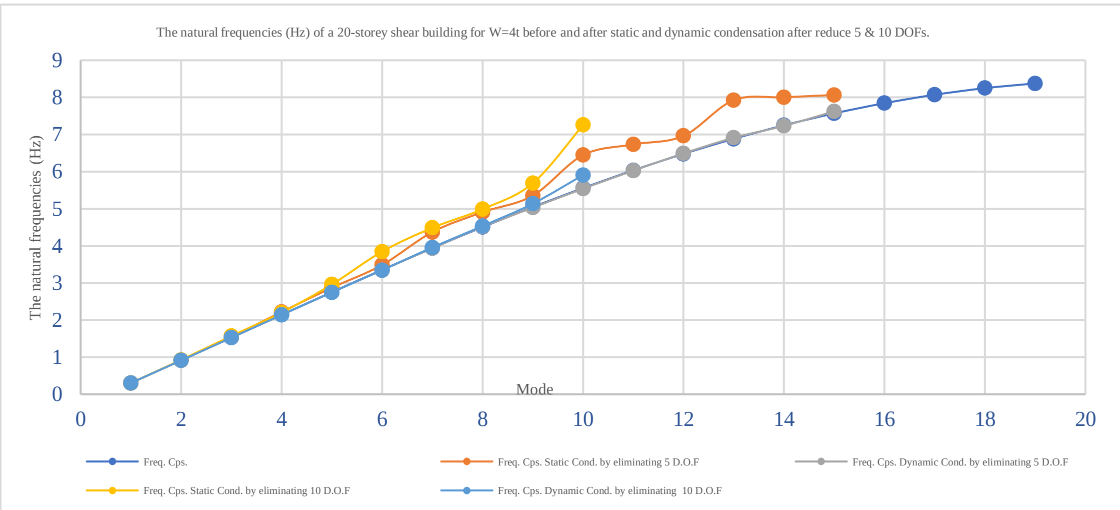
e



selecting the master degrees of freedom accurately once (keep the DOFs in which displacements are

Fig. 5 Comparison of frequencies when removing different numbers of degrees of freedom 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90%, respectively

accrued, and in which responses are of interest to), thus by selected masters DOFs should always be



displacements rather than rotations and another time randomly. Then the results were compared for 1st 10-Frequency as shown in the following Table. (3), (4), (5), (6), (7), (8), (9), (10) & (11) between the natural frequencies before and after the static and the dynamic condensation.

From the previous results, we find that the results of dynamic condensation are much more accurate than static condensation, especially in dynamic analysis. Also, the careful selection of the degrees of freedom of the master increases the accuracy, and most importantly, the greater the number of degrees of freedom removed, the more accuracy decreases until the results become very far from correct, as shown in Fig. (6).

TABLE 3. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 90% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining%90 D.O. F**	Error %	**Dynamic condensation by remaining %90D.O. F**	Error%	**Static condensation by remaining %90D.O. F random selection**	Error%	**Dynamic condensation by remaining %90D.O. F random selection**	Error%
1	**2.005**	**2.005**	0.0	**2.005**	0.0	**2.012**	0.349	**2.01**	0.249
2	**2.325**	**2.325**	0.0	**2.325**	0.0	**2.329**	0.1717	**2.327**	0.086
3	**2.377**	**2.377**	0.0	**2.377**	0.0	**2.381**	0.168	**2.376**	0.0421
4	**2.874**	**2.875**	0.0348	**2.874**	0.0	**2.877**	0.1044	**2.876**	0.06954
5	**4.445**	**4.45**	0.1124	**4.445**	0.0	**4.547**	2.2432	**4.444**	.0225
6	**4.839**	**4.877**	0.7792	**4.84**	0.0207	**5.012**	3.4517	**5.006**	3.3359
7	**5.437**	**5.531**	1.6995	**5.436**	0.0184	**5.626**	3.3594	**5.561**	2.2298
8	**6.021**	**6.09**	1.133	**6.019**	0.0332	**6.091**	1.14923	**6.091**	1.1492
9	**6.091**	**6.174**	1.3443	**6.09**	0.0164	**6.363**	4.2747	**6.123**	0.5226
10	**6.431**	**6.513**	1.259	**6.437**	0.0932	**6.682**	3.7563	**6.498**	1.0311

TABLE 4. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 80% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 80% D.O. F**	Error %	**Dynamic condensation by remaining 80%D.O. F**	Error %	**Static condensation by remaining 80%D.O. F random selection**	Error %	**Dynamic condensation by remaining 80%D.O. F random selection**	Error %
1	**2.005**	**2.006**	0.0499	**2.005**	0.0	**2.014**	0.4489	**2.005**	0.0
2	**2.325**	**2.326**	0.043	**2.325**	0.0	**2.332**	0.3	**2.325**	0.0
3	**2.377**	**2.377**	0.0	**2.377**	0.0	**2.382**	0.2099	**2.379**	0.0841
4	**2.874**	**2.875**	0.0348	**2.874**	0.0	**2.878**	0.138	**2.875**	0.0348
5	**4.445**	**4.478**	0.7369	**4.445**	0.0	**4.637**	4.14	**4.446**	0.0225
6	**4.839**	**5.087**	4.875	**4.867**	0.575	**5.179**	6.565	**4.912**	1.486
7	**5.437**	**5.842**	6.9326	**5.436**	0.0184	**5.635**	3.514	**5.444**	0.1286
8	**6.021**	**6.208**	3.0122	**6.021**	0.0	**6.215**	3.1215	**6.141**	1.954
9	**6.091**	**6.312**	3.5013	**6.113**	0.36	**6.431**	5.287	**6.231**	2.247
10	**6.431**	**7.047**	8.74	**6.698**	3.986	**7.23**	11.051	**6.823**	5.745

TABLE 5. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 70% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 70% D.O. F**	Error %	**Dynamic condensation by remaining 70%D.O. F**	Error%	**Static condensation by remaining 70%D.O. F random selection**	Error%	**Dynamic condensation by remaining 70%D.O. F random selection**	Error%
1	**2.005**	**2.006**	0.04985	**2.005**	0.0	**2.014**	0.4468	**2.005**	0.0
2	**2.325**	**2.326**	0.043	**2.325**	0.0	**2.332**	0.3002	**2.325**	0.0
3	**2.377**	**2.377**	0	**2.377**	0.0	**2.382**	.2099	**2.379**	0.0841
4	**2.874**	**2.875**	0.03478	**2.874**	0.0	**2.878**	0.1389	**2.875**	0.0348
5	**4.445**	**4.478**	0.7369	**4.444**	0.0225	**4.637**	4.1406	**4.446**	0.0225
6	**4.839**	**5.087**	4.87517	**4.898**	1.2046	**5.179**	6.565	**4.912**	1.4862
7	**5.437**	**5.842**	6.93256	**5.436**	0.0184	**5.635**	3.5138	**5.444**	0.1286
8	**6.021**	**6.208**	3.01224	**6.022**	0.0166	**6.215**	3.1215	**6.141**	1.9541
9	**6.091**	**6.312**	3.50127	**6.113**	0.3598	**6.431**	5.2867	**6.231**	2.2468
10	**6.431**	**7.047**	8.74131	**6.698**	3.9862	**7.23**	11.052	**6.823**	5.7453

TABLE 6. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 60**%** D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 60% D.O. F**	Error %	**Dynamic condensation by remaining 60%D.O. F**	Error%	**Static condensation by remaining 60%D.O. F random selection**	Error%	**Dynamic condensation by remaining 60%D.O. F random selection**	Error%
1	**2.005**	**2.008**	0.1494	**2.005**	0.0	**2.018**	0.6442	**2.005**	0.0
2	**2.325**	**2.33**	0.2146	**2.325**	0.0	**2.374**	2.064	**2.325**	0.0
3	**2.377**	**2.378**	0.0420	**2.377**	0.0	**2.527**	5.9359	**2.377**	0.0
4	**2.874**	**2.877**	0.1043	**2.874**	0.0348	**3.154**	8.8776	**2.876**	0.0695
5	**4.445**	**4.617**	3.7254	**4.446**	0.0225	**4.948**	10.165	**4.449**	0.0899
6	**4.839**	**5.416**	10.653	**4.921**	1.6663	**5.252**	7.8637	**4.978**	2.7922
7	**5.437**	**6.1**	10.8688	**5.421**	0.2951	**6.1**	10.858	**5.875**	7.4553
8	**6.021**	**6.406**	6.0099	**6.034**	0.2154	**6.432**	6.3899	**6.234**	3.4167
9	**6.091**	**6.516**	6.5224	**6.132**	0.6686	**6.967**	12.573	**6.294**	3.2253
10	**6.431**	**7.428**	13.4221	**6.722**	4.3291	**7.586**	15.225	**6.986**	7.9444

TABLE 7. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 50**%** D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining50% D.O. F**	Error %	**Dynamic condensation by remaining 50%D.O. F**	Error%	**Static condensation by remaining 50%D.O. F random selection**	Error%	**Dynamic condensation by remaining 50%D.O. F random selection**	Error%
1	**2.005**	**2.012**	0.347913	**2.005**	0.0	**2.018**	0.6483	**2.005**	0.0
2	**2.325**	**2.336**	0.47089	**2.325**	0.0	**2.338**	0.5591	**2.326**	0.0431
3	**2.377**	**2.379**	0.084069	**2.377**	0.0	**2.385**	0.3365	**2.377**	0.0420
4	**2.874**	**2.879**	0.173671	**2.875**	0.0347	**2.882**	0.2783	**2.875**	0.0349
5	**4.445**	**4.962**	10.41919	**4.444**	0.0225	**5.059**	13.813	**4.446**	0.0225
6	**4.839**	**5.647**	14.30848	**4.934**	1.9632	**5.405**	11.696	**4.936**	2.0045
7	**5.437**	**6.266**	13.23013	**5.413**	0.4414	**6.456**	18.742	**5.465**	0.5149
8	**6.021**	**6.546**	8.020165	**6.101**	1.3286	**6.733**	11.825	**6.145**	2.0594
9	**6.091**	**7.363**	17.27557	**6.432**	5.5984	**7.091**	16.417	**6.752**	10.852
10	**6.431**	**7.447**	13.64308	**6.633**	3.1410	**7.58**	17.866	**6.853**	6.5619

TABLE 8. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 40% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 40% D.O. F**	Error %	**Dynamic condensation by remaining 40%D.O. F**	Error%	**Static condensation by remaining 40%D.O. F random selection**	Error%	**Dynamic condensation by remaining 40%D.O. F random selection**	Error%
1	**2.005**	**2.018**	0.644202	**2.005**	0.0	**2.031**	1.2967	**2.007**	0.09975
2	**2.325**	**2.342**	0.725875	**2.325**	0.0	**2.38**	2.3656	**2.327**	0.08602
3	**2.377**	**2.385**	0.3354	**2.378**	0.0421	**2.399**	0.9255	**2.38**	0.12621
4	**2.874**	**2.886**	0.4158	**2.873**	0.0347	**2.937**	2.1920	**2.876**	0.06959
5	**4.445**	**5.08**	12.5	**4.447**	0.0449	**5.223**	17.502	**4.453**	0.17998
6	**4.839**	**6.116**	20.8797	**4.912**	1.5085	**6.521**	34.759	**5.092**	5.22835
7	**5.437**	**6.589**	17.4837	**5.765**	6.0327	**6.84**	25.804	**5.898**	8.47894
8	**6.021**	**7.038**	14.4501	**6.121**	1.6608	**7.099**	17.904	**6.421**	6.64341
9	**6.091**	**7.606**	19.9185	**6.583**	8.0775	**7.775**	27.647	**6.765**	11.0655
10	**6.431**	**7.653**	15.9676	**6.831**	5.1935	**8.144**	26.636	**6.876**	6.91961

TABLE 9. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 30% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 30% D.O. F**	Error%	**Dynamic condensation by remaining 30%D.O.F**	Error%	**Static condensation by remaining 30%D.O. F random selection**	Error%	**Dynamic condensation by remaining 30%D.O. F random selection**	Error%
1	**2.005**	**2.02**	0.74356	**2.005**	0.0	**2.213**	10.3741	**2.007**	0.09975
2	**2.325**	**2.347**	0.9374	**2.325**	0.0	**2.447**	5.24731	**2.328**	0.12903
3	**2.377**	**2.388**	0.46064	**2.379**	0.08414	**2.634**	10.8119	**2.376**	0.04207
4	**2.874**	**2.893**	0.65676	**2.876**	0.06959	**3.11**	8.21155	**2.879**	0.173974
5	**4.445**	**5.137**	13.4709	**4.448**	0.06749	**5.261**	18.3577	**4.45**	0.112486
6	**4.839**	**6.216**	22.1525	**4.841**	0.04133	**6.251**	29.1796	**4.845**	0.12399
7	**5.437**	**6.808**	20.1380	**5.764**	6.01434	**7.08**	30.2189	**5.876**	8.074305
8	**6.021**	**7.73**	22.1086	**6.143**	2.02624	**8.084**	34.2634	**6.345**	5.381166
9	**6.091**	**8.25**	26.1697	**6.713**	10.2118	**9.578**	57.2484	**6.813**	11.85355
10	**6.431**	**8.883**	27.6033	**6.801**	6.21987	**9.746**	51.5472	**6.912**	8.676722

TABLE 10. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 20% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 20% D.O. F**	Error%	**Dynamic condensation by remaining 20%D.O. F**	Error%	**Static condensation by remaining 20%D.O. F random selection**	Error%	**Dynamic condensation by remaining 20%D.O. F random selection**	Error%
1	**2.005**	**2.04**	1.71568	**2.006**	0.04987	**2.023**	0.88977	**2.007**	0.09975
2	**2.325**	**2.355**	1.27388	**2.326**	0.04301	**2.434**	4.47822	**2.327**	0.08602
3	**2.377**	**2.411**	1.41020	**2.378**	0.04207	**2.676**	11.1733	**2.379**	0.084139
4	**2.874**	**2.908**	1.16918	**2.876**	0.06959	**3.345**	14.0807	**2.867**	0.24356
5	**4.445**	**5.526**	19.5621	**5.049**	13.5883	**5.764**	22.8834	**5.457**	22.76715

TABLE 11. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 5- Story building by remaining 10% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 10% D.O. F**	Error %	**Dynamic condensation by remaining 10%D.O. F**	Error%	**Static condensation by remaining 10%D.O. F random selection**	Error%	**Dynamic condensation by remaining 10%D.O. F random selection**	Error%
1	**2.005**	**2.067**	2.9995	**2.008**	0.1496	**2.089**	4.18952	**2.01**	0.24937
2	**2.325**	**2.456**	5.3339	**2.378**	2.2796	**2.16**	7.09677	**2.389**	2.75268
3	**2.377**	**2.69**	11.6357	**2.884**	21.3294	**2.877**	21.0349	**2.986**	25.6205
4	**2.874**	**4.126**	30.3442	**3.654**	27.1399	**4.232**	47.2512	**3.875**	34.8295
5	**4.445**	**6.259**	28.9822	**6.123**	37.7503	**6.343**	42.6997	**6.201**	39.505

Example 4

The high-rise building configuration adopted is simple and regular, as shown in Fig. (7). A 30-story high-rise steel frame building with reinforced concrete slabs and brick walls is used as a simulation model, as shown in Fig. (8), as a case study. The steel frames are arranged in X and Z directions of the horizontal plane at a distance of 7 meters of the plot of land with a square perimeter (7x6m) in the X-direction and (7x6m) in the Z- direction. That is, a square area (42 x 42) m^2 as shown in Fig. (8). As for the steel frame's main resist gravity load as the main part and the lateral load as the secondary part, the height of the building was 90 meters, where it consisted of 30 stories, and the height of the story was H= 3 m.

A grade of steel S355 was used with Fy = 355MPa and Fu=480Mpa, where the beam sectors were selected with sectors IPE (500), whether it's inner or external beams, and the frame columns were divided into five groups, where the six below stories were with sectors H (400x990) and the next six stories with sectors H (400x634), the next six with sectors H (400x551), the next six with sectors H (400x383), and the last six columns set with H (400x237). Also, the steel bracing is used to resist the lateral loads as a pipe of the steel sections with a radius R= 21.9cm and thickness t = 5mm, as shown in Fig. (9). The reinforced concrete of the solid slab is with standard concrete grade C30/37, and it is a square slab with a cover weight = $0.15t/m^2$. The building mass is from the dead load, and the percentage is 25% from the live load as the SAP2000 solved by (Model Mode). The load for all beams is from their own weight, the wall weight, and the load from the slab, including the floor cover and the live load. In addition, the load for all columns is from its own weight, the wall weight, the beams load, and the load from the slab, including the floor cover and the live load. Created a FORTRAN program to generate the required data file for SAP2000 and the FORTRAN program for condensation. The natural frequency (cps) results were calculated using the FORTRAN program before and after the static and the dynamic condensation 90, 80, 70, 60, 50, 40, 30, 20, and 10%, respectively; this by selecting the master degrees of freedom accurately once (keep the degrees of freedom in which displacements are accrued, and in which responses are of interest to) and another time randomly in the selection of the masters of DOFs. Then the results were compared for 1st 10-Frequencies as shown in the following Table. (12), (13), (14), (15), (16), (17), (18), (19) & (20) between the natural frequencies before and after the static and the dynamic condensation.

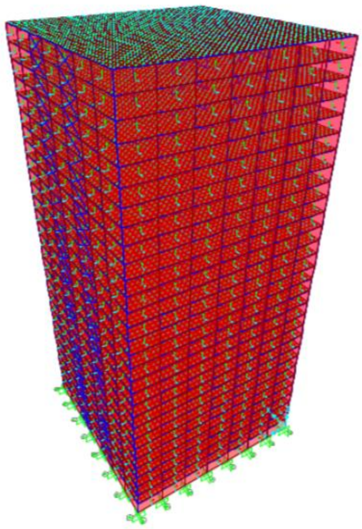


Fig.**10** A 30-story steel frame building.

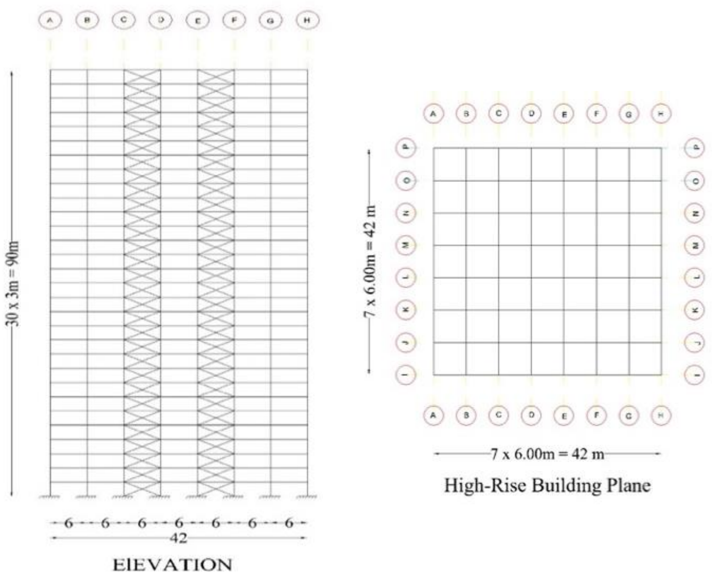


Fig.**9** A 30-storey steel frame building (42x42) with Hight= 30x3.0= 90.m.

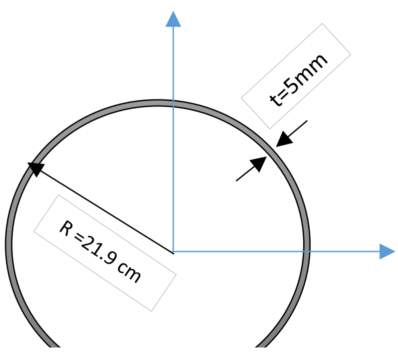


Fig. 7 Fig. 8. Pipe section R= 21.9 cm & t =5mm for Bracing sections.

TABLE 12. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30- Story building by remaining 90**%** D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining %90 D.O. F**	Error%	**Dynamic condensation by remaining %90D.O. F**	Error%	**Static condensation by remaining %90D.O. F random selection**	Error%	**Dynamic condensation by remaining %90D.O. F random selection**	Error%
1	**0.3465**	**0.3465**	0.0	**0.3465**	0.0	**0.3465**	0.0	**0.3465**	0.0
2	**0.3657**	**0.3657**	0.0	**0.3657**	0.0	**0.3657**	0.0	**0.3657**	0.0
3	**0.4464**	**0.4464**	0.0	**0.4464**	0.0	**0.4467**	0.06716	**0.4464**	0.0
4	**1.0039**	**1.0042**	0.0299	**1.0039**	0.0	**1.0051**	0.1194	**1.0037**	0.0199
5	**1.0755**	**1.0754**	0.0093	**1.0755**	0.0	**1.0761**	0.05576	**1.0778**	0.2134
6	**1.3121**	**1.3214**	0.7038	**1.3121**	0.0	**1.3341**	1.6491	**1.3133**	0.09137
7	**1.7302**	**1.7298**	0.0231	**1.7310**	0.0116	**1.7342**	0.2301	**1.7336**	0.1961
8	**1.869**	**1.8741**	0.2721	**1.8683**	0.0375	**1.9213**	2.7221	**1.8701**	0.0588
9	**2.2845**	**2.3013**	0.7300	**2.2912**	0.2837	**2.3432**	2.05051	**2.3011**	0.7214
10	**2.4701**	**2.4926**	0.9027	**2.4651**	0.2024	**2.5673**	3.7861	**2.4902**	0.8072

TABLE 13. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30- Story building by remaining 80**%** D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining %80 D.O. F**	Error%	**Dynamic condensation by remaining D.O.%80 F**	Error%	**Static condensation by remaining D.O.%80 F random selection**	Error%	**Dynamic condensation by remaining 80%D.O. F random selection**	Error%
1	**0.3465**	**0.3465**	0.0	**0.3465**	0.0	**0.3465**	0.0	**0.3465**	0.0
2	**0.3657**	**0.3657**	0.0	**0.3657**	0.0	**0.3658**	0.0273	**0.3657**	0.0
3	**0.4464**	**0.4464**	0.0	**0.4464**	0.0	**0.4468**	0.0895	**0.4465**	0.0224
4	**1.0039**	**1.0036**	0.0300	**1.0039**	0.0	**1.0061**	0.2187	**1.0043**	0.0398
5	**1.0755**	**1.0758**	0.0279	**1.0755**	0.0	**1.0871**	1.0671	**1.0781**	0.2412
6	**1.3121**	**1.3228**	0.8089	**1.3120**	0.0076	**1.3421**	2.2353	**1.3142**	0.1598
7	**1.7302**	**1.7401**	0.5689	**1.7308**	0.0347	**1.7412**	0.6317	**1.7423**	1.6937
8	**1.869**	**1.8727**	0.1975	**1.8701**	0.0588	**1.9654**	4.9048	**1.9012**	2.5467
9	**2.2845**	**2.3089**	1.0681	**2.2931**	0.3750	**2.4366**	6.2423	**2.3442**	2.547
10	**2.4701**	**2.5001**	1.2125	**2.4820**	0.4795	**2.6024**	5.0838	**2.5021**	1.2790

TABLE 14. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30- Story building by remaining 70**%** D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 70% D.O. F**	Error%	**Dynamic condensation by remaining 70%D.O. F**	Error%	**Static condensation by remaining 70%D.O. F random selection**	Error%	**Dynamic condensation by remaining 70%D.O. F random selection**	Error%
1	**0.3465**	**0.3465**	0.0	**0.3465**	0.0	**0.3465**	0	**0.3465**	0
2	**0.3657**	**0.3657**	0.0	**0.3657**	0.0	**0.3661**	0.1093	**0.3658**	0.0273
3	**0.4464**	**0.4464**	0.0	**0.4464**	0.0	**0.4541**	1.6957	**0.4466**	0.0448
4	**1.0039**	**1.0035**	0.0399	**1.0039**	0.0	**1.0071**	0.317	**1.0045**	0.0597
5	**1.0755**	**1.0763**	0.0743	**1.0755**	0.0	**1.0889**	1.2306	**1.0791**	0.3336
6	**1.3121**	**1.3278**	1.1965	**1.3124**	0.0229	**1.4232**	7.8064	**1.3154**	0.2509
7	**1.7302**	**1.7401**	0.5689	**1.7308**	0.0347	**1.8132**	4.5775	**1.7654**	1.9938
8	**1.869**	**1.8741**	0.2721	**1.8721**	0.1656	**2.0012**	6.6060	**1.9124**	2.2694
9	**2.2845**	**2.3174**	1.4197	**2.3012**	0.7257	**2.4532**	6.8767	**2.3956**	4.6377
10	**2.4701**	**2.5364**	2.6139	**2.4912**	0.847	**2.6142**	5.5122	**2.5524**	3.2242

TABLE 15. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30- Story building by remaining 60% D.O.F accurately and randomly.

	Frequencies (cps)							
	Exact Sol.by FORTRAN	**Static condensation by remaining 60% D.O. F**	Error%	**Dynamic condensation by remaining 60%D.O.F**	Error%	**Static condensation by remaining 60%D.O.F random selection**	Error%	**Dynamic condensation by remaining 60%D.O.F random selection**	Error%
1	**0.3465**	**0.3465**	0.0	**0.3465**	0.0	**0.3466**	0.0289	**0.3465**	0
2	**0.3657**	**0.3657**	0.0	**0.3657**	0.0	**0.3663**	0.16380	**0.3659**	0.0547
3	**0.4464**	**0.4464**	0.0	**0.4464**	0.0	**0.4544**	1.7605	**0.4467**	0.0672
4	**1.0039**	**1.0042**	0.0299	**1.0039**	0.0	**1.0083**	0.4364	**1.0046**	0.0697
5	**1.0755**	**1.0772**	0.1578	**1.0755**	0.0	**1.0893**	1.2669	**1.0812**	0.5272
6	**1.3121**	**1.3321**	1.5014	**1.3148**	0.2054	**1.4254**	7.9486	**1.3234**	0.8539
7	**1.7302**	**1.7467**	0.9446	**1.7315**	0.0751	**1.7933**	3.5186	**1.7892**	3.2976
8	**1.869**	**1.8782**	0.4898	**1.8723**	0.1763	**2.0224**	7.5851	**1.9434**	3.8283
9	**2.2845**	**2.3312**	2.0411	**2.3024**	0.7774	**2.4598**	7.1266	**2.4123**	5.2979
10	**2.4701**	**2.5532**	3.2547	**2.5053**	1.4444	**2.6156**	5.5628	**2.6012**	5.0399

TABLE 16. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30- Story building by remaining 50% D.O.F accurately and randomly.

	Frequencies (cps)							
	Exact Sol.by FORTRAN	**Static condensation by remaining 50% D.O. F**	Error%	**Dynamic condensation by remaining 50%D.O.F**	Error%	**Static condensation by remaining 50%D.O.F random selection**	Error%	**Dynamic condensation by remaining 50%D.O.F random selection**	Error%
1	**0.3465**	**0.3465**	0.0	**0.3465**	0.0	**0.347**	0.1441	**0.3466**	0.0289
2	**0.3657**	**0.3657**	0.0	**0.3657**	0.0	**0.3675**	0.4898	**0.3664**	0.19105
3	**0.4464**	**0.4462**	0.0448	**0.4464**	0.0	**0.4582**	2.5753	**0.4475**	0.2458
4	**1.0039**	**1.0048**	0.0896	**1.0038**	0.01	**1.0143**	1.0253	**1.0048**	0.0896
5	**1.0755**	**1.0783**	0.2597	**1.0755**	0.0279	**1.1093**	3.0469	**1.0832**	0.7109
6	**1.3121**	**1.3505**	2.8434	**1.3165**	0.3342	**1.4333**	8.4560	**1.3254**	1.0035
7	**1.7302**	**1.7521**	1.2499	**1.7385**	0.4774	**1.8012**	3.9418	**1.7932**	3.5133
8	**1.869**	**1.9120**	2.2490	**1.8802**	0.5957	**2.0353**	8.1708	**1.9454**	3.9272
9	**2.2845**	**2.3436**	2.5218	**2.3214**	1.2065	**2.5498**	10.4047	**2.4254**	5.8094
10	**2.4701**	**2.5765**	4.1063	**2.5123**	1.6559	**2.6756**	7.6805	**2.6244**	5.8794

TABLE 17. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30- Story building by remaining 40% D.O.F accurately and randomly.

	Frequencies (cps)							
	Exact Sol.by FORTRAN	**Static condensation by remaining 40% D.O. F**	Error%	**Dynamic condensation by remaining 40%D.O.F**	Error%	**Static condensation by remaining 40%D.O.F random selection**	Error%	**Dynamic condensation by remaining 40%D.O.F random selection**	Error%
1	**0.3465**	**0.3465**	0.0	**0.3465**	0.0	**0.3532**	1.8970	**0.3471**	0.1729
2	**0.3657**	**0.3655**	0.0547	**0.3657**	0.0	**0.3701**	1.18887	**0.3671**	0.3814
3	**0.4464**	**0.4467**	0.0672	**0.4464**	0.0	**0.4634**	3.6685	**0.4478**	0.3126
4	**1.0039**	**1.0052**	0.1293	**1.0041**	0.0199	**1.0461**	4.0340	**1.0053**	0.1393
5	**1.0755**	**1.0791**	0.3336	**1.0762**	0.0650	**1.1135**	3.4127	**1.0845**	0.8298
6	**1.3121**	**1.3568**	3.2945	**1.3181**	0.4552	**1.4255**	7.9551	**1.3655**	3.9107
7	**1.7302**	**1.7601**	1.6988	**1.7404**	0.5861	**1.8123**	4.5302	**1.8013**	3.9474
8	**1.869**	**1.9120**	2.2490	**1.8802**	0.5957	**2.0654**	9.5090	**1.9654**	4.9049
9	**2.2845**	**2.3678**	3.518	**2.3232**	1.6658	**2.5645**	10.9183	**2.4454**	6.5797
10	**2.4701**	**2.6165**	5.5953	**2.5608**	3.5419	**2.7334**	9.6327	**2.6624**	7.2228

TABLE 18. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30-Story building by remaining 30% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 30% D.O. F**	Error%	**Dynamic condensation by remaining 30%D.O.F**	Error%	**Static condensation by remaining 30%D.O.F random selection**	Error%	**Dynamic condensation by remaining 30%D.O.F random selection**	Error%
1	**0.3465**	**0.3461**	0.1156	**0.3466**	0.0289	**0.3598**	3.6965	**0.3452**	0.3766
2	**0.3657**	**0.3623**	0.9384	**0.3655**	0.0547	**0.3791**	3.5347	**0.3681**	0.6520
3	**0.4464**	**0.4423**	0.9269	**0.4461**	0.0672	**0.4744**	5.9022	**0.4482**	0.4016
4	**1.0039**	**1.0089**	0.49559	**1.0032**	0.0698	**1.0765**	6.7441	**1.0132**	0.9179
5	**1.0755**	**1.0809**	0.4996	**1.0789**	0.3151	**1.1257**	4.4594	**1.0923**	1.5380
6	**1.3121**	**1.3917**	5.7196	**1.3312**	1.4348	**1.4655**	10.467	**1.3689**	4.14932
7	**1.7302**	**1.7854**	3.0917	**1.7498**	1.1201	**1.8623**	7.0934	**1.8326**	5.5877
8	**1.869**	**1.9674**	5.0015	**1.9073**	2.0081	**2.1124**	11.5224	**1.9689**	5.0739
9	**2.2845**	**2.4492**	6.7246	**2.3642**	3.3711	**2.6245**	12.9548	**2.4445**	6.5453
10	**2.4701**	**2.8123**	12.1679	**2.6031**	5.1093	**3.1322**	21.1385	**2.6976**	8.4332

TABLE 19. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30-Story building by remaining 20% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 20% D.O. F**	Error%	**Dynamic condensation by remaining 20%D.O.F**	Error%	**Static condensation by remaining 20%D.O.F random selection**	Error%	**Dynamic condensation by remaining 20%D.O.F random selection**	Error%
1	**0.3465**	**0.3501**	1.0283	**0.3467**	0.0577	**0.3603**	3.8301	**0.3451**	0.4057
2	**0.3657**	**0.3609**	1.3300	**0.3658**	0.0273	**0.3823**	4.3421	**0.3683**	0.7059
3	**0.4464**	**0.4418**	1.0412	**0.4468**	0.0895	**0.4767**	6.3562	**0.4494**	0.6676
4	**1.0039**	**1.01**	0.6040	**1.0023**	0.1596	**1.0812**	7.1495	**1.0165**	1.2395
5	**1.0755**	**1.0987**	2.112	**1.0812**	0.5272	**1.1272**	4.5866	**1.0943**	1.7179
6	**1.3121**	**1.4013**	6.3655	**1.3345**	1.6785	**1.4768**	11.1525	**1.3721**	4.3728
7	**1.7302**	**1.8342**	5.6700	**1.7563**	1.4861	**1.8857**	8.2463	**1.8412**	6.0286
8	**1.869**	**2.0322**	8.0307	**1.9654**	4.9049	**2.1215**	11.9020	**1.9701**	5.1317
9	**2.2845**	**2.6533**	13.8998	**2.3722**	3.6970	**2.6437**	13.5870	**2.4463**	6.6140
10	**2.4701**	**2.9123**	15.1839	**2.6201**	5.7250	**3.2028**	22.8769	**2.7015**	8.5656

TABLE 20. Comparison between FORTRAN Solution Before Cond., After Static and Dynamic Condensations for 30-Story building by remaining 10% D.O.F accurately and randomly.

	Frequencies (cps)								
	Exact Sol.by FORTRAN	**Static condensation by remaining 10% D.O. F**	Error%	**Dynamic condensation by remaining 10%D.O.F**	Error%	**Static condensation by remaining 10%D.O.F random selection**	Error%	**Dynamic condensation by remaining 10%D.O.F random selection**	Error%
1	**0.3465**	**0.3561**	2.6959	**0.3463**	0.0578	**0.3612**	4.0697	**0.3492**	0.7732
2	**0.3657**	**0.3729**	1.9308	**0.3669**	0.3271	**0.3832**	4.5668	**0.3687**	0.8137
3	**0.4464**	**0.4515**	1.1296	**0.4481**	0.3794	**0.4784**	6.6889	**0.4502**	0.8441
4	**1.0039**	**1.023**	1.8671	**1.0139**	0.9863	**1.0931**	8.1602	**1.0175**	1.3366
5	**1.0755**	**1.1512**	6.5757	**1.1092**	3.0382	**1.1302**	4.8399	**1.1034**	2.5285
6	**1.3121**	**1.4346**	8.5389	**1.3357**	1.7669	**1.4823**	11.4822	**1.3743**	4.5259
7	**1.7302**	**1.9112**	9.4705	**1.7721**	2.3644	**1.8932**	8.6097	**1.8527**	6.6119
8	**1.869**	**2.1205**	11.8604	**2.0012**	6.6060	**2.1256**	12.0718	**1.9821**	5.7061
9	**2.2845**	**2.7087**	15.6606	**2.3891**	4.3782	**2.6866**	14.9668	**2.4501**	6.7589
10	**2.4701**	**3.0134**	18.029	**2.6213**	5.7681	**3.2276**	23.4694	**2.7121**	8.9229

4. RESULTS AND DISCUSSION

A general discussion clarifies what was proposed in this current study to understand the advantages of different condensation methods during the free analysis of high-rise buildings.

All results, whether before or after condensation, were from free analysis to determine the natural frequencies. The dynamic properties of the structural systems are obtained by solving the eigenvalue problem. The results before condensation are verified with the results determined by the finite element program. The results of the dynamic condensation show good agreement with a maximum deviation of ± 5:7%.

Analysis using static and dynamic condensation was conducted, and the results were compared with the before condensation case. Table 1. shows the comparison between the FORTRAN solution before condensation and after applying the static and dynamic condensations for the shear building a case (4DOFs). The mass is constant, and all DOFs are horizontal, so no master DOFs are required. The results show that the first frequency deviates by about 2.24% and 10.56% for the second frequency in static condensation. While in the dynamic condensation, the results were improved where the frequency deviates by about 0.02% and 5.43% for the first and second frequencies, respectively, compared to the solution before condensation. Table 2. Presents a comparison between the FORTRAN solution before condensation and after using the static and dynamic condensations with a reduction of 5 and 10 DOFs for a 20-story shear building. The results of dynamic condensation are much better than static condensation, whether when removing 5 DOFs or 10 DOFs. Moreover, the condensed model with removed 5 DOFs is more accurate than that with removed 10 DOFs. The reason is eliminating more DOFs decreases the accuracy of the model; it is preferable to keep a more significant number of DOFs as a master. The results of comparing the FORTRAN solution without condensation and after applying the static and dynamic condensations for a 5-story building in 3D with 120 DOFs are presented in Tables 3 to 11. The study began by decreasing the degrees of freedom by 10% and continued to reduce the degrees of freedom by 10% until it reached 90% of the total number of degrees of freedom removed. For the static condensation, the random selection of the master DOFs leads to an error reaching the maximum of 15.2% with a 40% reduction in DOFs; please see Table 6. The careful selection of the master DOFs increased the accuracy, and the error reached only 13.4%. However, the dynamic condensation proved more accurate in any case, with deviations of only 4.3:7%. For all methods, when the reduction of DOFs increases by 50%, the solution becomes imprecise and unreliable. The results of a reduction of 80% or 90% of DOFs in Tables 10 and 11. are completely incorrect because much of the main DOFs were dispensed with.

In case 4, the same condensation approach was followed, and the results show the same behavior as the third case. The results of comparing the FORTRAN solution before condensation and after applying the static and dynamic condensations for a 30-story high-rise building are presented in Tables 12 to 20. When solving the 30-story building model, the finite element method without condensation takes more than 4 hours to show the required frequencies. At the same time, the created FORTRAN program for condensation takes only ten minutes to obtain the frequencies with a reduction of DOFs by 50%. It is worth mention to that the time decreases by increasing the condensation percentages.

5. CONCLUSIONS

The current research presents proposed static and dynamic condensation techniques for the free vibration analysis of high-rise buildings. Several results of the study can be summarized as follows.

The results revealed that dynamic condensation is more efficient in obtaining the frequencies and mode shapes than static condensation because the dynamic condensation methods consider the effects of inertia of ignored DOFs. The dynamic condensation for analyzed models gives a maximum deviation of ± 5:7% compared to the case before condensation.

Also, careful selection of the master DOFs increased the accuracy of the frequencies in the static condensation. However, the dynamic condensation demonstrated more accurate in any case. Moreover, the results proved that the greater the number of neglected degrees of freedom, the lower the degree of accuracy of the model.

The FORTRAN program, created to condense the matrices for dynamic problems, gives high-accuracy output while saving computer time, effort, and storage capacity.

6. CONFLICT OF INTEREST

The authors have no financial interest to declare in relation to the content of this article.

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