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NONLINEAR IMPEDANCE FUNCTIONS OF A SQUARE FOUNDATION PLACED ON SEMI-INFINITE SOIL

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ABSTRACT

The goal of this study is to find out how nonlinear soil behavior affects the dynamic impedance of a rigid surface foundation that is put under a harmonic dynamic load. Researchers can use a computational code that combines the boundary element method (BEM) and the thin layer method (TLM) to calculate the nonlinear dynamic impedances (stiffness and damping) of foundations in the frequency domain. This method integrates nonlinear soil behavior into a linear framework. The soil's nonlinear behavior is evident in the changes observed in its dynamic properties, specifically the decrease in the normalized shear modulus (G/Gmax), the increase in the normalized hysteretic damping coefficient (ξ/ξmax), and the variation in the equivalent shear wave velocity (Vs/Vsmax) as a result of the seismic excitation applied to the soil, which is dependent on the unit shear strain (γ/γr). The parameters are derived using the equivalent linear approach employed in the Caldynasoil computational code for various levels of seismic loading. We conducted a comprehensive analysis to assess how soil non-linearity affects the behavior of a soil-foundation system under various conditions. This included considering factors such as the soil's homogeneity or heterogeneity, the shape of the foundation, and whether the foundation was supported by a semi-infinite soil or constrained by bedrock. The study also considered different levels of seismic deformation, using five seismic accelerations ranging from 0.01g to 0.4g as reference points. The linear and nonlinear dynamic impedance coefficients of the massless foundation were determined for all vibration modes (translation, rocking, and torsion) as a function of the dimensionless excitation frequency a_0.

KEYWORDS: Dynamic impedance, Nonlinear soil, Equivalent linear method, BEM, TLM, Frequency domain.

دوال الممانعة غير الخطية لأساس مربع موضوع على تربة شبه لا نهائية

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الملخص

الهدف من هذه الدراسة هو معرفة كيف يؤثر سلوك التربة غير الخطية على الإعاقة الدينامية لقاعدة سطحية جامدة توضع تحت حمولة ديناميكية متناغمة. ويمكن للباحثين أن يستخدموا رمزا حسابيا يجمع بين طريقة عنصر الحدود (BEM) وطريقة الطبقة الرقيقة (TLM) لحساب معوقات ديناميكية غير خطية (الشفرة والسد) للأسس في مجال التردد. تدمج هذه الطريقة سلوك التربة غير الخطية في إطار خطي. ويتجلى سلوك التربة غير الخطي في التغيرات الملاحظة في خصائصها الدينامية، وعلى وجه التحديد الانخفاض في معامل المقص المطّرد (G/Gmax)، والزيادة في معامل التثبيط الهستيري المطّرد (ymax/1.00)، والتباين في السرعة المكافئة لموجة القص (Vs/Vssmax) نتيجة للإثارة الزلزالية المطبقة على التربة، التي تعتمد على سلالة مقص الوحدة (1.00 / ميكروغرام). وتُستمد البارامترات باستخدام النهج الخطي المكافئ المستخدم في الرمز الرقمي Caldynasoil لمختلف مستويات التحميل السيزمي. وقد أجرينا تحليلاً شاملاً لتقييم كيفية تأثير عدم تجانس التربة على سلوك نظام تأطير التربة في ظل ظروف مختلفة. وشمل ذلك النظر في عوامل مثل تجانس التربة أو عدم تجانسها، وشكل الأساس، وما إذا كان الأساس مدعوماً بتربة شبه منفية أو مقيداً بحجر الأساس. ونظرت الدراسة أيضا في مستويات مختلفة من التشوهات السيزمية، باستخدام خمسة تسارعات سيزمية تتراوح بين g0.0 و g0.4 كنقاط مرجعية. وقد حُدِّدت معاملات الإعاقة الدينامية الخطية وغير الخطية للمؤسسة التي لا كتلة لها بالنسبة لجميع أنماط الاهتزاز (التحويل، والارتكاز، والتصغير) كدالة لتردد الاستثارة a_0 .

الكلمات المفتاحية : المعاوقة الديناميكية، التربة غير الخطية، الطريقة الخطية المكافئة، BEM، TLM، مجال التردد.

INTRODUCTION

Soil-structure interaction is a branch of applied mechanics that focuses on developing and studying theoretical and practical techniques for analyzing structures under dynamic loads. It considers the behavior of both the soil and the foundation. The dynamic impedance term, also known as stiffness, plays a crucial role in the seismic analysis of a structure using the substructure approach. A key parameter heavily relies on the properties of both the foundation and the soil. References [1-10] conducted studies to estimate the dynamic impedances. However, all of these researches only examined the behavior of soil as either a linear elastic or viscoelastic substance. Researchers have conducted only a limited number of studies in the context of nonlinearity. The study conducted by [11] investigated the impact of soil nonlinearity on the dynamic behavior of soil-structure interactions, that had been achieved this by utilizing a numerical code that employed the finite element approach and incorporated the Iwan model. [12] investigated soil-structure interaction in a similar manner, considering the non-linear behavior of the soil.

The experimental tests conducted by [13] for granular materials and [14] for fine materials determined the nonlinear behavior of the soil. However, researchers can numerically calculate these two dynamic parameters for certain soil types in Algeria using the computational code FLAC, as described in [15]. Furthermore, researchers [16, 17] developed Caldynasoil, a computational code based on the equivalent linear method, to estimate the seismic response of a soil profile under acceleration applied at the bedrock level.

An equivalent linear technique is used in the substructure approximation to determine the dynamic response of the soil-foundation system when it exhibits nonlinear behavior. In a way similar to the thin-layer method (TLM), this method works by dividing the soil into flat layers, each with its own shear and damping moduli. Each soil sub-layer encompasses the fluctuations in the dynamic characteristics of the material consistent with each deformation caused by the seismic loading.

Changes in the dynamic properties of the soil, such as the reduction of the normalized shear modulus G/Gmax, the increase of the normalized hysteretic damping

ξ/ξ_{max}, and the variation of shear wave velocity amplitude V_s/V_{smax}, typically indicate the phenomenon of soil nonlinearity. The excitation applied to the soil, specifically in response to unit shear strain ($\gamma/\gamma r$), causes these changes. The behavior is simulated using the one-dimensional Caldynasoil computational code, which employs the equivalent linear method with concentrated masses and hyperbolic models [16 - 18].

The FanvibWave [10] is a three-dimensional computational code that we developed. The FanvibWave [10] is a three-dimensional computational code that we developed, which combines the boundary element method with thin layer theory (BEM-TLM). By integrating the linear-equivalent method procedure for nonlinear soil behavior, the code allows for the computation of nonlinear dynamic impedances at the interface of the soil-foundation system in the frequency domain.

A comprehensive three-dimensional parametric analysis demonstrated the impact of soil nonlinearity on a rigid surface foundation. This analysis considered both homogeneous and heterogeneous soil conditions, as well as semi-infinite or bedrock-bounded soil. Additionally, the study examined the effects of the foundation's shape.

The study determines the nonlinear impedance functions at the interface between the soil and foundation for all vibration modes. These results are then compared to the linear case, taking into account the a-dimensional frequency a_0. Five distinct seismic accelerations of peak ground acceleration (PGA) are employed to quantify the nonlinearity of the soil.

2. Soil-foundation system nonlinear behavior calculation

Three methods are employed to examine the interaction between soil and structure in the linear elastic range: direct, substructure, and hybrid. The direct method simultaneously determined the rest of the ground and structure in a single step. In contrast, the substructure method determined the response separately, using multiple steps. The hybrid method divided the ground-structure system into two subdomains, a far field and a near field, and determined their respective responses.

This work incorporates a novel equivalent linear methodology into the substructure method for the purpose of assessing foundation nonlinearity. This technique ascertained the nonlinear dynamic stiffness of the interface between the soil and the foundation, commonly referred to as the impedance function. The impedance K (6, 6) is linked to the dynamic load P (6, 1) and displacement U (6, 1) in the following manner:

$$P(\omega) = K(\omega).U(\omega) \quad (1)$$

In this situation, the foundation has no mass; hence the impedance represents the proportion of force that is directly transmitted to the foundation (which is the same as the response of the earth) due to the resulting displacement. The foundation is rigid and possesses six degrees of freedom. The defining feature of this is an impedance matrix K (ω) with dimensions of (6×6). The impedance matrix is contingent upon the frequency of the harmonic excitation, as expressed by the subsequent equation:

$$K(\omega)=\begin{bmatrix} K_{11}(\omega) & 0 & 0 & 0 & K_{1,5}(\omega) & 0 \\ 0 & K_{22}(\omega) & 0 & K_{2,4}(\omega) & 0 & 0 \\ 0 & 0 & K_{33}(\omega) & 0 & 0 & 0 \\ 0 & K_{4,2}(\omega) & 0 & K_{44}(\omega) & 0 & 0 \\ K_{5,1}(\omega) & 0 & 0 & 0 & K_{55}(\omega) & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{66}(\omega) \end{bmatrix} \quad (2)$$

The symbol ω represents the circular frequency of the harmonic vibration, whereas K (ω) represents the impedance functions matrix with dimensions of 6×6. The foundation's impedance coefficients, represented as Kij(ω), fluctuate depending on the vibration modes.

Fig. 1 illustrates the computational model. Considering a sturdy, cubic foundation over a consistent, boundless, and compacted bedrock. The soil possesses distinct characteristics, such as its bedrock elevation (Ht), compactness (ϱ), resistance to deformation (G), ability to dissipate energy (β), and Poisson's ratio (v). The foundation is subjected to three harmonic external loads, specifically Px, Py, and Pz, as well as three harmonic moments, specifically Mx, My, and Mz, which cause stress. Ultimately, determine the dynamic impedance coefficients of the foundation for each vibration mode. The soil and foundation are linked by the compatibility criterion, which can be mathematically represented as follows:

$$\{u\} = [R] \times \{D\} \quad (3)$$

with

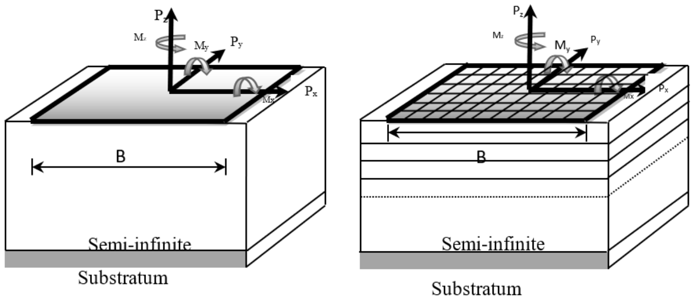
{u}: represents the displacements of soil;

[R]: the transfer matrix of dimension (6N×3) is denoted as follows:

$$[R]=\begin{pmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 0 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{pmatrix} \quad (4)$$

N: represents the quantity of elements present at the interface between the soil and the foundation;

{D}: indicates the foundation's displacements.



(a) (b)

Fig. 1. (a) The geometry of the model (b) The discretization of the model.

The load vector applied to the foundation is represented by {P}. The equilibrium between the load and the forces (traction) distributed over the elements that divide the volume of the foundation is expressed by the following relationship:

$$\{P\} = [R^T] \times \{t\} \quad (5)$$

{t}: represents the tensile values at the interface between the soil and foundation.

The relation between the soil displacements and the tractions distributed over the elements is expressed by:

$$\{u\} = [G] \times \{t\} \quad (6)$$

[G]: The flexibility matrix of the soil (Green function).

Combining equations (3), (5) and (6) we obtain the following equation:

$$\{P\} = ([R^T] \times [G]^{-1} \times [R]) \times \{D\} = [K(\omega)] \times \{D\} \quad (7)$$

where

$$[K(\omega)] = ([R^T] \times [G]^{-1} \times [R]) \quad (8)$$

$[K(\omega)]$: represents the dynamic impedance of the foundation.

Equation 7 provides a means to express the dynamic impedance of a rigid foundation for any degree of freedom.

$$K = k + i\omega C \quad (9)$$

Where k is the dynamic stiffness and $(\omega.C)$ is the stiffness loss. It can also be written as :

$$[K(a_0)] = [K_{st} \times (k(a_0) + i a_0 \times c(a_0))] \times (1 + i2\beta) \quad (10)$$

Where, K_{st} is the static stiffness; $k(a_0)$ stiffness coefficient; $c(a_0)$ damping coefficient; β is the hysteretic damping coefficient of the soil and $a_0 = (\omega \times B_x / C_s)$ is the dimensionless excitation frequency with B_x is the half width of the foundation and Cs is the velocity of the shear wave.

The goal of this paper is to examine the dynamic non-linear behavior of the foundations, taking into account the non-linear properties of the soil obtained by the Caldynasoil code [16] and [17]. The soil demonstrates nonlinearity when exposed to substantial shear deformation, such as intense seismic loading. The relationship between stress and the decrease in shear modulus (G/Gmax) and percentage of critical damping (ξ/ξ_{max}) can be utilized to characterize the behavior of soil under stress. Each layer in the model is represented as linear and elastic, with a concentrated mass. Figure 2 illustrates the fundamental mechanical model and the related soil profile, in which the soil mass is uniformly distributed both above and below each layer.

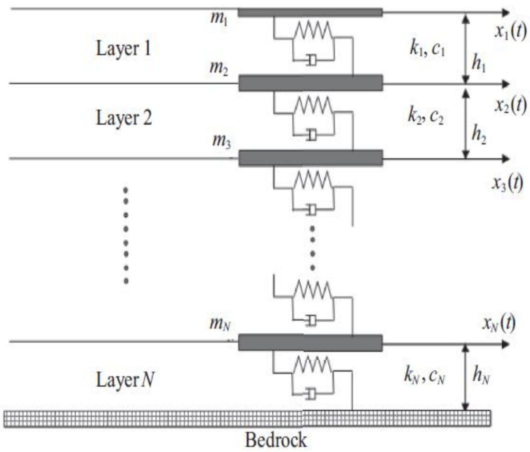


Fig. 2. The model with concentrated masses.

where,

$$m_1 = \frac{\rho_1 h_1}{2}\text{, and } m_i = \frac{\rho_{i-1} h_{i-1} + \rho_1 h_i}{2} \quad (11)$$

m_i, ρ_i and h_i is the concentrated mass, density and thickness of the soil layer i (= 2, 3…*N*).

3. RESULTS AND DISCUSSION

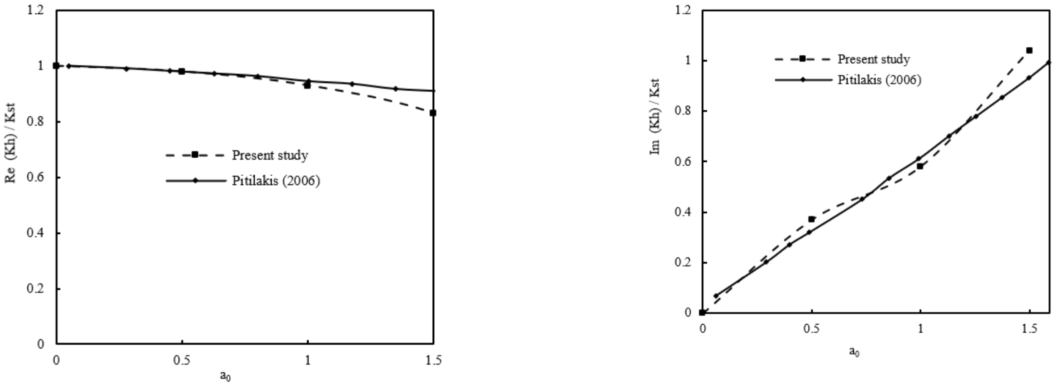
In this analysis, the response of the soil-foundation system in the nonlinear behavior studied. First, to find the nonlinear behavior of the soil by the Caldynasoil computational code [16] and [17], then to introduce the nonlinear properties of the soil into the FonvibWave computational code developed to obtain the nonlinear impedance functions of the foundation (the coefficients of rigidities and damping at the interface of the soil-foundation system).

3.1. Validation

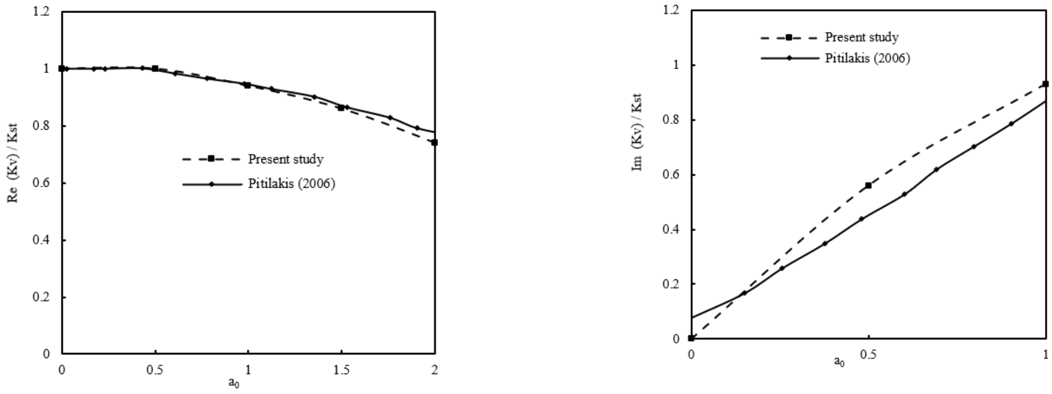
The impact of nonlinear soil behavior on the dynamic response of soil-structure interaction is not well established in this sector. Therefore, we can only verify our work based on the linear impedance scenario described in reference [19]. The validation of this study involves comparing the results produced by [19] while calculating the linear impedance functions using the boundary element method (BEM) in the frequency domain.

The foundation has a circular shape with a diameter of 10 meters. It is placed on the surface of a semi-infinite medium and is subjected to unit dynamic loads of Px = Py = Pz = 1 and unit moments of Mx = My = Mz = 1. The soil exhibits the following characteristics: the bedrock has a height of 30 m, simulating a semi-infinite condition. Its Poisson's ratio is 0.33, the coefficient of hysteretic damping is 0.02, the shear velocity is 180 m/s, and the density is 2000 kg/m^3.

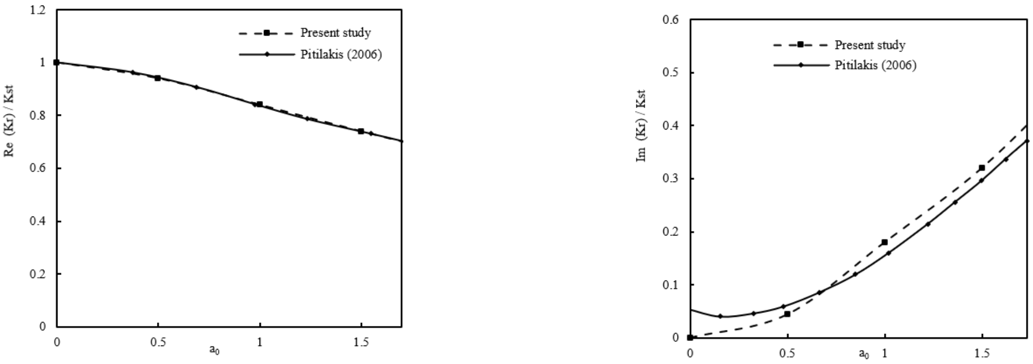
The validation involves determining the dimensionless impedance coefficients as a function of the dimensionless frequency a_0 for all vibration modes. **Fig. 3** demonstrates a strong correlation between the findings of this investigation and the results reported in reference [19]. The vibrational modes of both techniques exhibit nearly identical behavior across all frequencies.



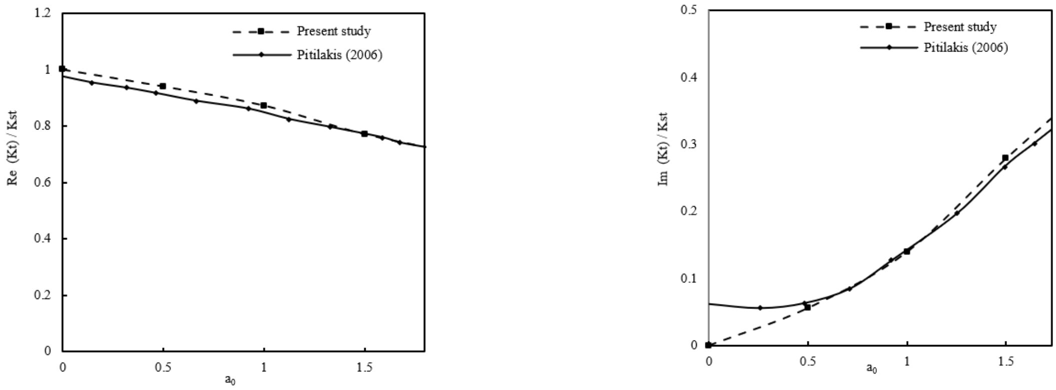
(a) Horizontal mode



(b) Vertical mode



(c) Rocking mode



(d) Torsion mode

Fig. 3. Validation for all modes of vibration.

3.2. Parametric analysis

The parametric study aimed to investigate the influence of soil nonlinearity on the dynamic impedance of a foundation. The study examined multiple scenarios, encompassing foundations of varying geometries (square or rectangular) and supported by either uniform or diverse soil. The foundations were either limited by bedrock or extended indefinitely. The calculations were conducted utilizing the three-dimensional FonvibeWave algorithm. The user's text pertains to a particular source or citation. The Caldynasoil computational code, developed by Filali and Sbartai in 2012, was utilized to perform a one-dimensional (1D) simulation. The simulation employed the equivalent linear technique and Massing's (1926) hyperbolic model to examine the non-linear response of the soil to different earthquake deformations. The foundation is subjected to unit forces and unit moments, with each component having a value of 1. These forces and moments create stress on the foundation. The foundation soil in the linear elastic case possesses the following attributes: Ht = 10m (representing a semi-infinite medium), density ϱ = 1, Poison's coefficient v = 0.333, maximum shear modulus Gmax = 1, and initial damping coefficient ξmax = 0.05. Fig. 4 depicts the nonlinear behavior using five seismic accelerations that were recorded: Creek-0.01g, Loma Prieta-Diamond-0.1 g, Nahanni-0.2g, El Centro-0.3 g, and Los Angeles-0.4 g. The seismic forces provide dimensionless numbers that depict the non-linear dynamic properties of the soil. The values are determined by measuring the amplitude of deformation and are recorded in Table 1.

Table 1 presents the lower and upper limits of shear modulus, and damping coefficient for each seismic strain (γ/γ_r) , (G/G_{max}), and (ξ/ξ_{max}), respectively. Figure 5 illustrates the variations of two moduli $(G/G_{max}, \xi/\xi_{max})$ in response to seismic distortions, with the peak ground acceleration (PGA) ranging from 0.01g to 0.4g.

Table 1. Variations in nonlinear dynamic soil properties with earthquake acceleration level.

Accelerogram(s)	(γ/γ_r)	G/G_{max}	ξ/ξ_{max}
Linear	0.0001	1	0.05
a_{max} = 0.01g	0.014	0.96	0.06
a_{max} = 0.1g	0.75	0.58	0.18
a_{max} = 0.2g	3.28	0.24	0.45
a_{max} = 0.3g	9.74	0.10	0.66
a_{max} = 0.4g	31.41	0.03	0.84

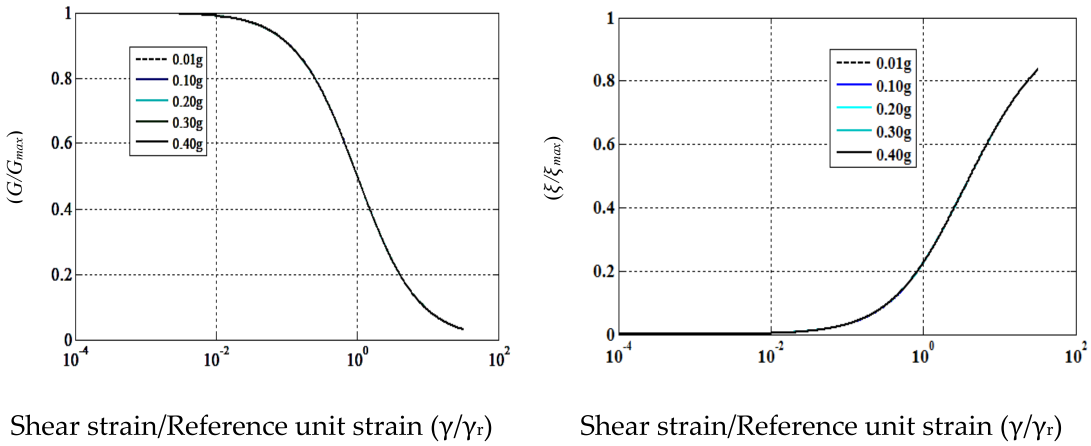
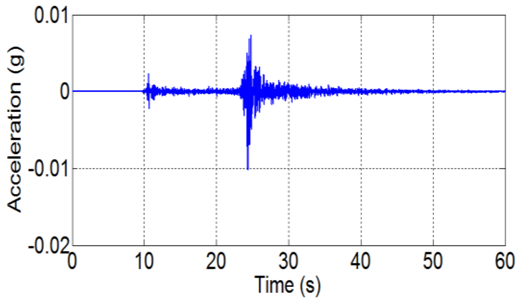
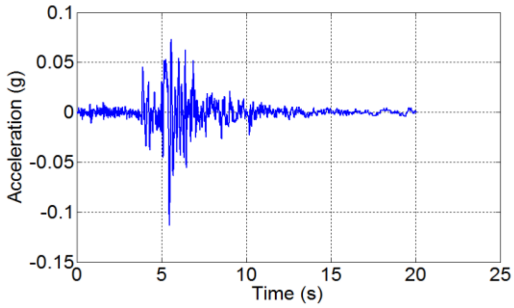


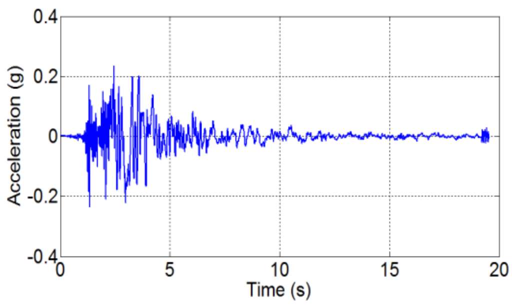
Fig. 4. **S**hear modulus ratio and damping ratio as a function of shear strain.



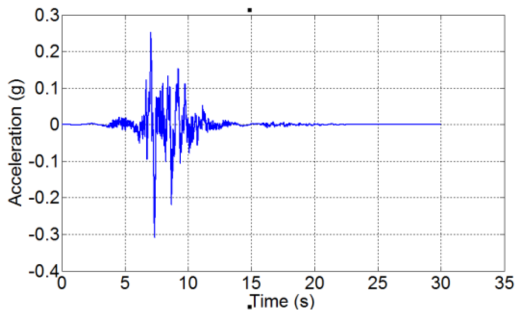
(a) The acceleration time histories of the Creek, Loma Prieta, 2016 (0.01g)



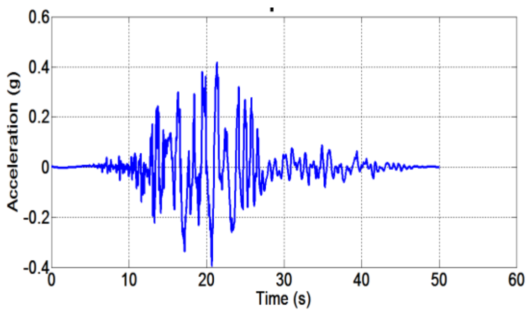
(b) The acceleration time histories of the 1989 (0.1g)



(c) The acceleration time histories of the Nahanni, 1985 (0.20g)



(d) The acceleration time histories El-Centro1940 (0.30g)



(d) The acceleration time histories of the Los Angeles,1989 (0.4g)

Fig. 5. The five Strong motion records.

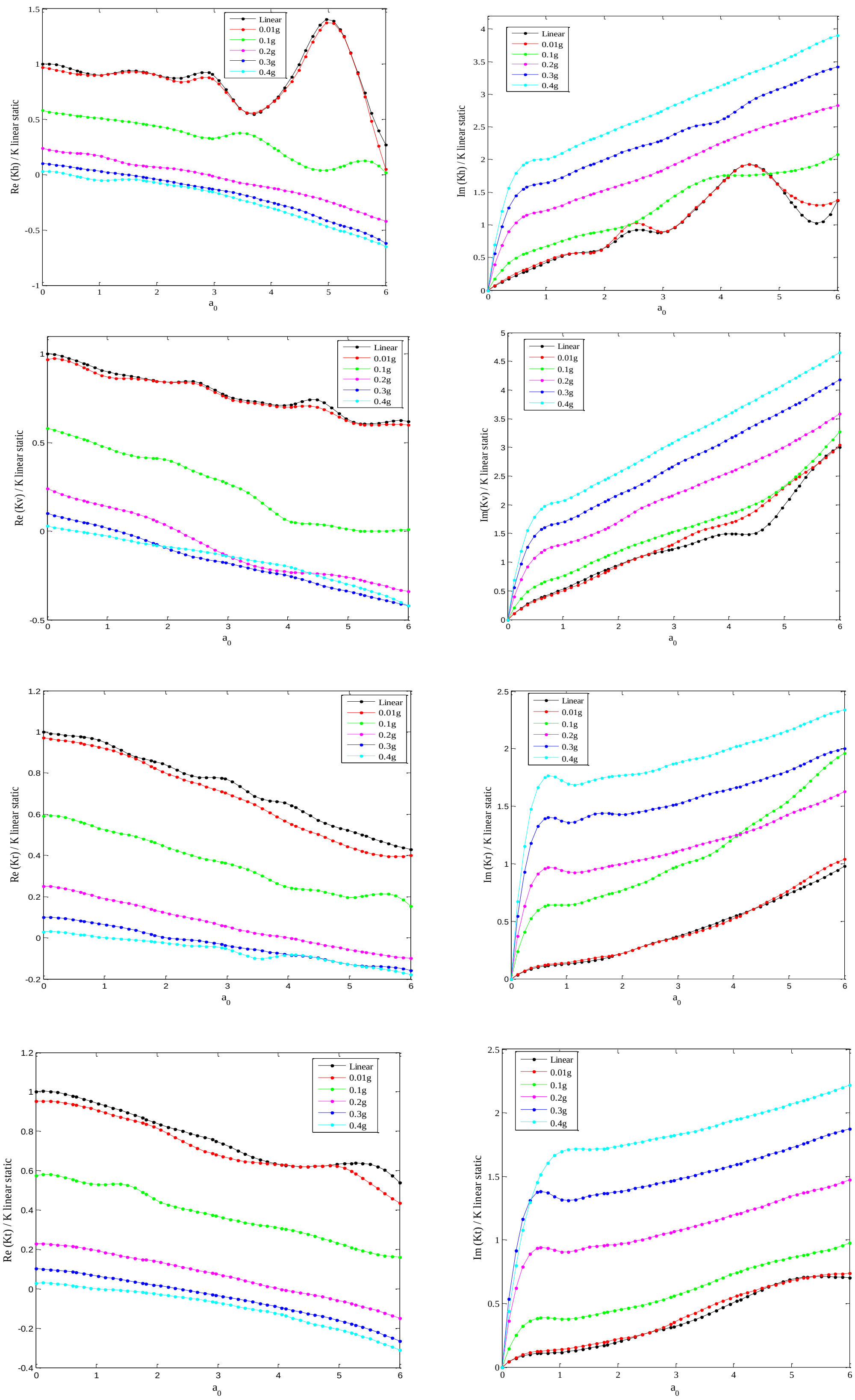


Fig. 6. Linear and nonlinear equivalent dynamic impedance of a square foundation on a semi-infinite homogeneous soil.

Conclusions

This study investigates the impact of soil's nonlinear behavior on the dynamic response of soil-foundation interaction. An extensive study was conducted to illustrate how the non-linear behavior of soil affects the dynamic impedance coefficients (stiffness and damping) of a rigid surface foundation placed on uniform or non-uniform soil, whether it is infinite or bound by rock. The study's findings suggest that:

- The soil exhibits nonlinear behavior, which is characterized by alterations in its dynamic properties when subjected to seismic deformation.
- The changes consist of a reduction in the shear modulus G/G_{max}, an augmentation in the damping coefficient ξ/ξ_{max}, and modifications in the velocity of the shear wave.
- The non-linearity of the ground fluctuate depending on the magnitude of seismic acceleration.
- The dynamic non-linear impedance coefficients (stiffness and damping) of the foundation demonstrate a significant correlation with the soil behavior and the dimensionless excitation frequency a_0.
- The real component of the impedance, which signifies the stiffness coefficient, declines, while the imaginary component, which signifies the damping coefficient, increases as the distortion caused by the seismic load intensifies in all vibration modes.
- A study examining the influence of the non-linear properties of the soil on the impedance of a foundation placed on a uniform soil surface indicates that this phenomenon impacts all modes of the foundation's impedance, especially at higher frequencies. Starting from the deformation of PGA = 0.2 g, it is evident that there is a substantial decrease in stiffness and a more noticeable increase in the damping coefficient of the impedance.

This study offers a comprehensive insight into the response of the soil-foundation system to non-linear behavior, specifically when subjected to strong seismic vibrations that result in reduced stiffness of both the soil and the foundation.

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