



## STOCHASTIC OPTIMIZATION OF ELECTRIC GENERATION SYSTEMS CONSIDERING INPUT DATA UNCERTAINTIES

Mohamed Ahmed Hassan El-Sayed\*

Electric Power Department, Faculty of Engineering, Cairo University

\*Correspondence: [elsmah@hotmail.com](mailto:elsmah@hotmail.com) [elsmah@eng.cu.edu.eg](mailto:elsmah@eng.cu.edu.eg)

### Citation:

M.A.H. El-Sayed, "Stochastic optimization of electric generation systems considering input data uncertainties", Journal of Al-Azhar University Engineering Sector, vol. 19, pp. 316 - 332, 2024.

Received: 20 January 2024

Revised: 09 May 2024

Accepted: 27 May 2024

DOI:10.21608/auj.2024.264258.1593

Copyright © 2024 by the authors.  
This article is an open-access article distributed under the terms and conditions of Creative Commons Attribution-Share Alike 4.0 International Public License (CC BY-SA 4.0)

### ABSTRACT

To achieve greenhouse gas neutrality, the electric utilities need to integrate large amounts of intermittent renewable energy sources (RES). This integration results in high demand for energy exchange from the liberalized market according to the surpluses production or storage options. Classical generation planning assumes that the input data are deterministic, which leads to an increase in the risk potential due to the fluctuation range of this data. At the present stage, most of Generation planning techniques considering the uncertainties of input variables focus on Monte Carlo (MC) simulation and artificial neural networks (ANNs). However, MC and ANNs require comprehensive computation facilities and a big data base and also need problem-dependent modification or even integration with other techniques. These limitations make it challenging to achieve the economic operation of large-scale systems with future and spot market energy. Therefore, this paper presents integrated planning algorithm based on stochastic consideration of the uncertain input data such as the predicted consumer load, the solar radiation, wind speed, the electricity prices on the exchanges in liberalized markets depending mainly on scenario analysis. Thereby, the optimization problem is decomposed into multi-stage decision-making process based on depicting the uncertainties in scenarios, each of which is weighted with its probability of occurrence. In this scenario analysis, the objective function consists of minimizing the annual cost over the entire scenario tree. Due to the high demands on computing time and storage space in practical systems, decomposition approach based on Lagrange relaxation is used in this paper for solving the stochastic optimization problem. Finally, the simulation results show that the proposed stochastic optimization significantly enhances generation under high degree of uncertainties in input data.

**KEYWORDS:** Data uncertainties, Generation systems, Stochastic optimization, Scenario tree, Decomposition of large systems.

## التحسين العشوائي لتوليد الكهرباء مع الأخذ في الاعتبار عدم اليقين في بيانات الإدخال

محمد أحمد حسن السيد\*

قسم القوى الكهربائية، كلية الهندسة، جامعة القاهرة.

\*البريد الإلكتروني: [elsmah@hotmail.com](mailto:elsmah@hotmail.com) [elsmah@eng.cu.edu.eg](mailto:elsmah@eng.cu.edu.eg)

### الملخص

تحقيق حياض الغازات الدفيئة، تحتاج الأنظمة الكهربائية إلى دمج كميات كبيرة من مصادر الطاقة المتجددة. ويؤدي هذا التكامل إلى ارتفاع الطلب على تبادل الطاقة من السوق المحررة وفقاً للفوائض المتاحة. وبالتالي، ينبغي تنفيذ القرارات المستقبلية للحصول على التوليد المثالي من أجل الإمداد الاقتصادي للطلب على الكهرباء. ويفترض حالياً في تخطيط التوليد الكلاسيكي أن البيانات المدخلة معروفة بالضبط، مما يؤدي إلى زيادة احتمالية المخاطر بسبب نطاق تقلب هذه البيانات. معظم تقنيات التخطيط التي تأخذ في الاعتبار أوجه عدم اليقين في متغيرات المدخلات تركز على محاكاة مونت كارلو والشبكات العصبية الاصطناعية. ومع ذلك، فهي تتطلب مرافق حسابية شاملة وقاعدة بيانات كبيرة وتحتاج أيضاً إلى التعديل المعتمد على المشكلة أو حتى التكامل مع التقنيات الأخرى. وتجعل هذه القيود من الصعب تحقيق التشغيل الاقتصادي لوحدات التوليد والطاقة في السوق الفورية. لذلك،

يقدم هذا البحث خوارزمية تخطيط متكاملة تعتمد على الاعتبار العشوائي لبيانات المدخلات غير المؤكدة مثل حمل المستهلك المتوقع، والإشعاع الشمسي، وسرعة الرياح، وأسعار الكهرباء في الأسواق المحررة. وتعتمد هذه الخوارزمية العشوائية بشكل أساسي على تحليل السيناريوهات، والذي يعرف مشكلة التحسين بأنها عملية اتخاذ قرار متعددة المراحل تعتمد على تصوير أوجه عدم اليقين في التطور المستقبلي في سيناريوهات، يتم وزن كل منها باحتمالية حدوثها. في تحليل السيناريو هذا، تتكون وظيفة هدف التحسين من تقليل التكلفة السنوية عبر شجرة السيناريو بأكملها. نظرًا للمتطلبات العالية لوقت الحوسبة ومساحة التخزين في الأنظمة العملية، تم استخدام أسلوب التلحل المعتمد على استرخاء لاغرانج في هذا البحث لحل مشكلة التحسين العشوائي. وأخيرًا، أظهرت نتائج المحاكاة أن التحسين العشوائي المقترح يعزز بشكل كبير عملية التوليد في ظل درجة عالية من عدم اليقين في البيانات المدخلة.

**الكلمات المفتاحية :** عدم اليقين في البيانات، أنظمة التوليد، التحسين العشوائي، شجرة السيناريو، تقسيم الأنظمة الكبيرة

## 1. INTRODUCTION

To reduce greenhouse gas emission, the electric utilities need to integrate large amounts of intermittent renewable energy sources (RES). This integration results in high demand for energy trading with the liberalized market according to surpluses or production bottlenecks. Consequently, the future optimal decisions should integrate generation and trading planning for economic supply of the electric demand. The classical generation planning assumes that the input data are deterministic and exactly known, which leads to an increase in the risk potential due to the fluctuation range of this data [1]. Therefore, an integrated planning algorithm based on stochastic consideration of the uncertain input data such as the predicted consumer load, the solar radiation, wind speed, the electricity prices on the exchanges in liberalized markets has to be developed. Thereby, the flexibility of the developed algorithm should reflect the operational and technical ability of the generating system to respond to deviations in the input data from their predicted expected values [2].

In previous operational practice, the uncertain input data are simulated in generation planning via their expected values. Assuming that the effect of remaining uncertainties can be handled by maintaining minute reserves to react to unforeseen events. In developing countries, planning based on deterministic approach can no longer be applied unreservedly. On the one hand, these countries are currently experiencing an increase in consumer loads that is difficult to predict. On the other hand, the composition of the available power plant park is changing due to the increased use of regenerative generation systems such as wind energy converters, whose performance and energy contribution to the electrical energy supply is dependent on the availability of weather data and is therefore difficult to be accurately forecasted.

MC simulation is a general-purpose, simple-to-implement method for uncertainty propagation. MC technique has been applied in domains outside the game such as planning, scheduling, and operation optimization under uncertainty [3]. However, in more complex or large-scale systems an efficient application of this technique often requires comprehensive computation facilities. Moreover, by implementing Monte Carlo it can be difficult to properly assign probability distributions to uncertain input variables [3].

At present, ANNs have been used for energy analysis. It was important to gather training and validation big datasets and comprehensive computing facilities for applying ANN in practical generation systems. Among those networks is backpropagation neural network (BPNN), using a backpropagation algorithm. However, BPNNs are limited in their ability to solve specific small-scale systems. Since a BPNN corrects network connection weights using the root mean square error

(RMSE) and gradient descent technique, unavoidable issues such as slipping into local minima, sluggish convergence speed, and overfitting may exist [4].

Since the development of the simplex algorithm by George Dantzig, linear programming (LP) has become a standard procedure for all kinds of optimization problems [5]. Numerous improvements of the original algorithm have been developed to enhance its performance and to incorporate additional requirements for the feasibility of solutions such as integer linear programming, Dantzig-Wolfe-Decomposition and Benders Decomposition, Lagrangian Relaxation. The last approach has received an enormous amount of attention recently due to its usefulness in many practical settings, such as stochastic optimization, and has found wide applications in the engineering problems.

Economic generation is one of the most significant optimization operation problems that has attracted a high number of researchers [6]. The economic generation can be defined as finding the least power generation costs from conventional generating units or renewable power plants to satisfy the demand. At the same time, it satisfies the system constraints of the generation limits and system power reserve. \*One approach for economic generation in practical large-scale systems are decomposition techniques, which aim to solve a large optimization problem by repeatedly solving smaller and simpler subproblems, while guaranteeing that the final solution is optimal for the original problem [7]. The subproblems can be solved efficiently by avoiding the original system inherent complexities and computational challenges.

A suitable stochastic approach is proposed in this paper based on scenario analysis, which interprets the optimization problem as a multi-stage decision process [8]. The basic principle depends on depicting the uncertainties of future development in scenarios, each of which is weighted with its probability of occurrence. In this way, the original problem with its input data described by the distribution function, is broken down into many deterministic sub-problems, which span the scenario tree and in total have similar statistical properties to the original problem. This means the result of each single sub-problem is the optimization of certain section in this scenario tree. A key task is the appropriate coordination of the use of power plants at the branching points in the scenario tree. In scenario analysis, the optimization objective function consists of minimizing the the generation cost over the entire scenario tree. In the very near future, the input information is associated with low uncertainties due to high forecast accuracy and is therefore virtually unambiguous. As the planning horizon increases, the forecast uncertainties increase, so that the number of different scenario sections increase.

Based on the above review analysis, this paper proposes a multi-stage decision-making process based on depicting the uncertainties in scenarios tree and applying decomposition approach coordinated by Lagrange relaxation for solving the stochastic optimization problem in practical system with the following inherent advantages:

- The proposed algorithm facilitates the estimation of the economic generation cost under uncertain input variable.
- The required computation time and storage space are acceptable for solving practical systems based on the suggested scenario tree and decomposition technique.
- The dynamic process of the proposed Lagrange multipliers enhances the solution of generation planning by quick convergence.
- The effect of water inflows and the uncertainty of wind and solar are considered using proposed scenario tree.

- The suitable number of scenario segments can be selected to reduce the output error, and calculation time.

The paper is organized as follows. After introduction Section 2 makes a brief presentation of problem statement and objectives. Section 3 presents system analysis and modelling. Section 4 describes the applied optimization approach beginning with the generation of scenario tree and end by optimization procedure. Section 5 presents the studied generation system. Section 6 discusses the obtained numerical results and their statistical distributions. Finally, Section 7 presents the conclusions and highlights the main findings of this research with suggestions for future work.

## 2. PROBLEM STATEMENT AND OBJECTIVES

The main objectives of this paper are summarized as follows:

- analysis of the relevant input variables, which are uncertain.
- generation of the scenario tree with the corresponding probabilities of its segment's occurrence.
- minimization of the annual generation costs, utilizing the stochastic processes of generation planning.
- development of the appropriate algorithm for solving practical generating system including renewable, future and spot market energy.
- decomposition of the optimization problem based on Lagrangian relaxation to fulfill the technical constraints of the studied generation system.
- quantification of the effect of input variables uncertainties on annual generation cost.
- derivation of fundamental findings from this stochastic optimization of generation planning including uncertain input data and how these findings may change the planning results.

## 3. SYSTEM ANALYSIS AND MODELLING

The studied system consists mainly of thermal and hydraulic power plants. The operation of conventional power plants is influenced by their technical generation limits and the cost of input fuel. Short-term electricity trading can be used for economic transactions [9, 10]. At the system boundary, the effect of the spot market on the generation system is considered because of its short-term trading activities.

Generation planning requires a load forecast in the daily range. The load forecasts are less accurate the further they are directed into the future. The deviations between the predicted and actual load approximately follow the normal distribution. The relative standard deviation of a load forecast is between 4 and 7% on a daily average with a maximum value of 10% [10].

### 3.1. Thermal power plant units

Nowadays, the largest proportion of the electrical energy demand is covered by thermal power plants that are fired with fossil fuels. A power plant usually consists of several blocks that can be operated independently. The operating costs of a thermal power plant are determined by fuel consumption, the fluctuating primary energy price and additional operation and maintenance costs. The heat consumption depends non-linearly on the power generated and can be approximated with accepted accuracy using a second-order polynomial between the maximum and minimum generated power. The amount of the spinning reserve contribution of each unit results from the difference between the maximum capacity and the actual generated power. In addition, blocks that

can be started quickly have the option of contributing to the minute reserve. Minimum operating times and minimum downtimes are defined for the individual blocks to avoid too frequent start-ups and shutdowns. The outage of thermal blocks influences power plant operation and is described by the failure frequency and failure duration [14].

### **3.2. Hydraulic power plants**

In general, a hydraulic generation system can be divided into storage, pump storage, and run-of-river power plants. The regenerative water supply is available without variable costs. In contrast to thermal power plant blocks, the available power plant output and the usable electrical energy are directly influenced by the current water inflow. As ancillary conditions, the mandatory water quantities for land irrigation must be observed. The amount of water flowing into a hydraulic power plant system is subject to large seasonal and stochastic fluctuations. The annual cycle of the inflow amounts is determined by the climatic conditions. The stochastic annual fluctuations lead to the water supply being particularly high in one year (wet year) and particularly low in another (dry year). The complete description of the stochastic water inflow is then determined by its probability density function for the study period [11].

### **3.3. Electricity trading**

By the liberalization of the electricity industry, electrical energy is traded over short periods of time on spot markets or over longer periods of time on futures markets. In spot the delivery of electrical energy follows the conclusion of the contract immediately. Within the trading model, the difference between the purchase price and the selling price must be considered. Statements on the forecast accuracy of the spot market price in the daily range can only be found to a very limited extent in the literature. For example, the hydraulically dominated Scandinavian market, forecasting methods of spot price show error amounts between 8 and 19% depending on the time of day [12]. For the pool in GB, the mean error using artificial neural networks is estimated as 12 to 18%, depending on their topologies.

### **3.4. Renewable wind and solar resources**

Generation planning requires an hourly forecast of the feed-in from wind turbines. The power output is simulated by conversion using a generation characteristic of the individual wind turbines. The shading effects that typically occur in large scale wind farm are recorded by a characteristic curve that is modified compared to an individual wind turbine. The wind speed time series obtained from the superimposition of the short-term fluctuations and the long-term series is then converted into a power time series using the power characteristic of the committed wind turbines. Forecasting methods show errors with a typical standard deviation of 15% when estimating the power output of large-scale wind farms [13]. The forecast errors are generally approximated by normal distribution and increase with increasing forecast horizon.

Generally, there is a large seasonal variation in global horizontal irradiance (GHI) up to more than  $1000\text{Wm}^{-2}$  in summer and less than  $300\text{Wm}^{-2}$  in winter. Accurate prediction of solar irradiance is valuable for solar generation in energy markets. The clouds have a great effect on the solar radiation reaching the Earth's surface. Therefore, accurate cloud forecasts are essential for prediction of solar radiation. For performing the solar radiation forecast, the recorded data of the previous five years are used to train the applied RNN. For the studied system, the estimated standard deviation in solar radiation forecast is equal to 18% [20]. As the percentage sharing of

RES is gradually increased in the proposed planning horizon, therefore the reserve margin is annually increased to cope with the RES uncertainty effect. The reserve margin, which represents the difference between the generation capacity and the demand forecasted, is maintained higher than 15% in annual generation planning.

### 4. OPTIMIZATION APPROACH

Uncertain input data is mapped as described in Fig. 1 using scenario analysis [15]. Considering many scenarios would ensure that the obtained generation plane is robust against uncertain influences in the future.

#### 4.1. Generation of the scenario tree

An approach for generating scenario trees is used to carry out generation planning under uncertainty of the input variables. For each period corresponding to a scenario tree branch, the distribution density function is determined from the forecasted value of the uncertain input variables. This function is then subdivided into classes so that each sub-areas of the distribution density function correspond to a class (e.g.: high, medium, and low input value). If information about temporal correlations of the uncertain variables is available, the conditional probabilities between the classes can be applied for the considered periods. If this information is not available, the variables are assumed to be temporally uncorrelated, i.e. the transitions between the classes of consecutive time segments are assumed to be equally probable, Fig. 1. Starting from the first period, which is considered certain, the scenario tree can then be built up step by step by tracing all possible transitions. Since the size of the scenario tree grows exponentially with the number of branching points, only those scenarios whose transition probability exceeds a given specified value are further considered.

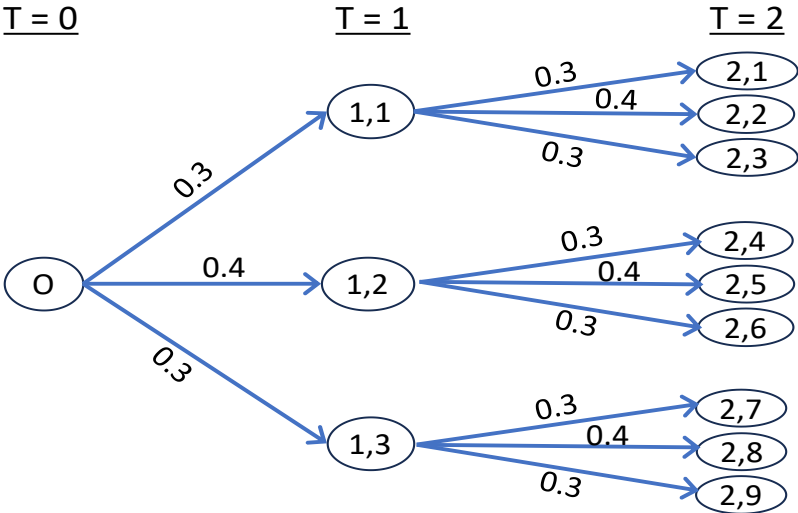


Fig. 1. Modeling of uncertain input variables using scenarios.

In each scenario section (s), however, the capacity of committed generation must be always guaranteed with the total load plus reserve balance. Moreover, in the power balance, the generated and traded electrical power corresponds to the system load  $P_{s,t}^L$  at all time intervals  $t = 1, 2 \dots T$ .

$$\sum_{m=1}^{N_{th}} P_{s,t}^{th,m} + \sum_{j=1}^{N_{hy}} P_{s,t}^{hy,j} + \sum_{k=1}^{N_{wi}} P_{s,t}^{wi,k} + \sum_{i=1}^{N_{so}} P_{s,t}^{so,i} + P_t^{Sp} = P_{s,t}^L \quad (1)$$

This power balance has a system-wide coupling effect and links the load sale  $P_{s,t}^L$ , the thermal  $P_{s,t}^{th}$  and hydraulic generation  $P_{s,t}^{hy}$ , the feed-in from wind turbines  $P_{s,t}^{wi}$ , the feed-in from solar thermal systems  $P_{s,t}^{so}$  and spot trading  $P_t^{Sp}$ .  $N_{th}$  denotes the number of thermal blocks,  $N_{hy}$  the number of hydraulic power plants,  $N_{wi}$  and  $N_{so}$  the number of different wind and solar units. The reserve requirement  $P_{s,t}^R$  couples the operated thermal and hydraulic generating units as follows:

$$\sum_{m=1}^{N_{th}} (P_{s,t}^{th,m,max} - P_{s,t}^{th,m}) + \sum_{j=1}^{N_{hy}} (P_{s,t}^{hy,j,max} - P_{s,t}^{hy,j}) \geq P_{s,t}^R \quad (2)$$

## 4.2 Procedure

In the existing planning techniques, a distinction is made between methods for the global optimization task of the large-scale generation systems and methods that are based on decomposition techniques. In principle, methods of linear programming (LP), quadratic programming (QP) and mixed integer linear programming (MILP) can be used for the global solution. Due to the high demands on computing time and storage space, these methods are not able to solve stochastic optimization tasks on the considered size [16]. For this reason, a decomposition approach is chosen to solve the optimization problem in this paper as shown in Fig. 2. Starting with the scheduled exchange and economic decision of the zoned energy, the different generation units are optimized against the trade markets.

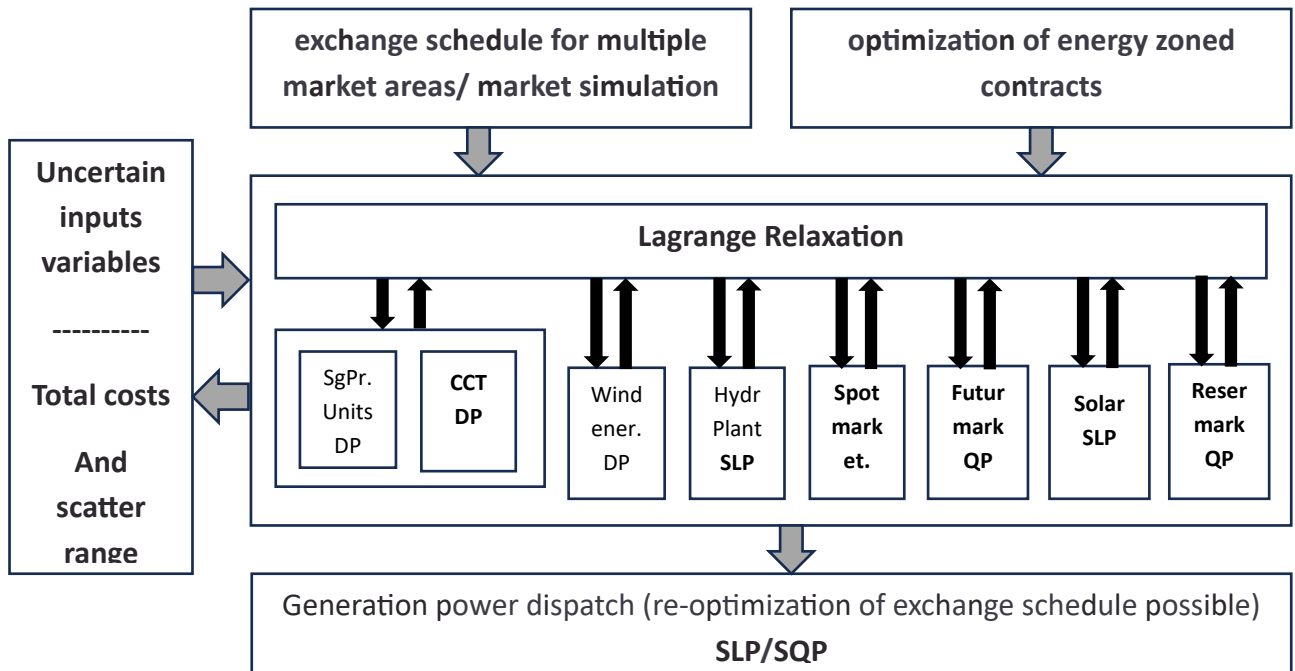


Fig. 2. Overview of the integrated optimization approach.

Decomposition approaches divide the global optimization tasks into smaller subtasks to then be solved independently. With decomposition approaches, there is an additional effort to ensure that the linking constraints between these subtasks are fulfilled. There are basically two ways of decomposition: decomposition in the system domain and decomposition in the time domain. In the first option, the task is broken down into the optimization of the individual subsystems, i.e., the generation of the individual units and trading on the spot market are determined independently. A coordinator ensures compliance with the system-technical constraints. The system-wide coupled power and reserve balance is only guaranteed by the coordinator, to whom the individual optimization modules are subordinate with equal rights [17]. For arranging the different optimization modules below the coordinator, the Lagrangian decomposition represents a suitable method for decomposing the overall problem. For breakdown in the time domain, individual time intervals or individual time sections are optimized independently from the rest of the planning period. In this case, there is an additional coordination effort to ensure compliance with the time-coupling conditions due to the minimum times of the thermal blocks and the reservoir levels. This will then have a negative effect on the convergence behavior of this method [18].

The division of generation subsystem to thermal, hydro, RES, future and spot markets enhances their operation planning by utilizing the appropriate optimization methodologies according to the technical characteristics of each subsystem. This serves first to elevate the application of DP optimization for thermal and wind generation. Second, this offers Successive Linear Programming (SLP) for hydro and solar optimization. Economic decision of future, spot market and reserve contracts is carried out using the QP. Finally, this study offers the hydro-thermal dispatch using the SLP and Sequential Quadratic Programming (SQP) as displayed in **Fig. 2**.

The procedure of the proposed optimization takes place in two steps. First, all possible decisions of the power plants and the amounts of energy traded on the spot market are determined by the iterative coordination process in the Lagrange relaxation [19]. Based on these results, the feasible solutions that fulfill the load and reserve balance are economically evaluated. Thereby, the objective function (OF) is minimizing the generation costs ( $K_{ges}$ ) from conventional power plants, wind or solar units and spot trading as follows:

$$OF = Min[(K_{ges}^{th}) + (K_{ges}^{hy}) + (K_{ges}^{so}) + (K_{ges}^{wi}) + (K_{ges}^{sp})] \quad (3)$$

The calculated value of the total generation cost and electricity trading in each scenario sections (s) should be weighted with its probability of occurrence Prs. Therefore the resulted objective function (OF) of the optimization task is reformulated by:

$$OF = Min \left[ \sum_s Prs \{ (K_{ges}^{th}) + (K_{ges}^{hy}) + (K_{ges}^{so}) + (K_{ges}^{wi}) + (K_{ges}^{sp}) \} \right] \quad (4)$$

Using the Lagrangian decomposition, the objective function of the primal problem of equation (4) is extended to the dual problem by relaxing the system-wide coupling boundary conditions. Lagrange multipliers  $\lambda_{s,t}$  and  $\mu_{s,t}$  are introduced for the power and reserve balance condition resulting in the following augmented dual Lagrange function:

$$L(\lambda_{s,t}, \mu_{s,t}) = \sum_s Prs \{ [(K_{ges}^{th}) + (K_{ges}^{hy}) + (K_{ges}^{so}) + (K_{ges}^{wi}) + (K_{ges}^{sp})] +$$



$$\lambda_{s,t} \left[ \sum_{m=1}^{N_{th}} P_{s,t}^{th,m} + \sum_{j=1}^{N_{hy}} P_{s,t}^{hy,j} + \sum_{k=1}^{N_{wi}} P_{s,t}^{wi,k} + \sum_{i=1}^{N_{so}} P_{s,t}^{so,i} + P_t^{Sp} - P_{s,t}^L \right] + \mu_{s,t} \left[ \sum_{m=1}^{N_{th}} (P_{s,t}^{th,m,max} - P_{s,t}^{th,m}) + \sum_{j=1}^{N_{hy}} (P_{s,t}^{hy,j,max} - P_{s,t}^{hy,j}) - P_{s,t}^R \right] \quad (5)$$

The problem is solved by iterative adjustment of the Lagrange multipliers  $\lambda_{s,t}$  and  $\mu_{s,t}$ , so that when the method converges, the power and reserve balances are maintained. Starting with initial value for  $\lambda_{s,t}$  and  $\mu_{s,t}$ , the objective function value and its sub gradients are determined to update the values of these multipliers. The procedure is repeated until the multiplier's values converge with those of the respective preceding iteration. The quantities that can be traded on the spot market are assumed to be a continuous variable, where positive and negative value correspond to purchase to and sale of electrical power on the spot market, respectively.

## 5. THE STUDIED GENERATION SYSTEM

The generation system contains a renewable energy capacity of 6 GW. The total available capacity from the thermal power plants equals 25702 MW with 128 units. The forecasted peak load in summer due to air conditioning demand reaches 23211 MW. The cost of thermal generating units consists of fixed and variable costs in addition to starting and shutting down costs.

About 85% of the energy-related generation costs are made up of fuel costs. The remaining small part of these energy-related costs consists of the provision of auxiliary materials and maintenance dependent on the operating time. The dependency of fuel expenses in a power plant unit is described by a parabola, while the additional energy-related costs are assumed to be proportional to the supplied electrical energy. The thermal generation subsystem is supplemented by five hydroelectric power plants, where about 7% of the annual electrical energy is covered by hydraulic power plants.

The measured values of the direct solar radiation and wind speed are used for simulating solar and wind generation. Since the optimization process is based on a time raster of one hour, the series of the direct radiation and wind speed are also generated in hourly intervals and converted into the relevant power output via the technical characteristics of the solar panels and wind turbines. Network and total load are thus again modeled as super node. The maximum tradable quantity for purchase and sale is set at 600 MW per product. The market price fluctuations reflect the daily, weekly, and seasonal fluctuations in electrical energy demand as well as the cycles in supply-dependent generation. The expected value of the assumed hourly price fluctuates between 15 and 40 €/MWh over the course of the day. From the perspective of annual generation planning, the price of short-term electricity deals in the future is uncertain and needs to be forecasted. Methods of time series analysis and market simulation models are used to determine market price forecasts and show that the standard deviation of hourly electricity prices is around 15%.

## 6. NUMERICAL RESULTS

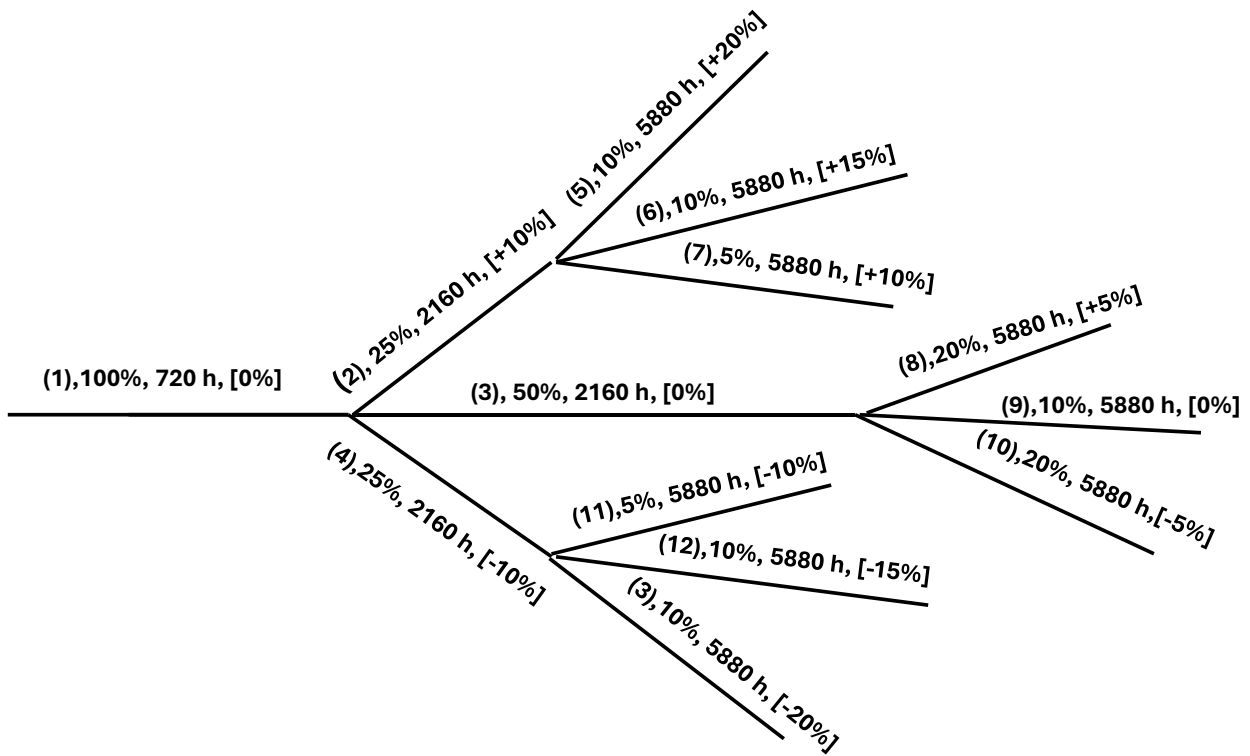
To study the effects of the uncertainty, the optimization results are compared with the deterministic calculations assuming that all input variables are certain and fixed at their mean

forecasted values. The most important criterion for comparing the different calculations is the expected value of the generation cost with its scatter range. All calculations were performed on Sun Fire computing station with Ultra Spac III processors (900 MHz) under Solaris 9.0. The total computing time for annual calculation was almost 10 hours with a memory requirement of 616 MB. The duality gap, an upper estimate of the distance from the theoretically achievable optimum, is in the per thousand range in relation to the total generation cost for all studied cases.

**Table 1.** Overall results of the deterministic calculation.

Total energy sales	154.728 TWh
Energy consumption	142.844 TWh
Sale spot market	11.884 TWh
Thermal generation	134.613 TWh
Hydraulic generation	11.605 TWh
Purchase spot market	31.953 TWh
Wind energy	4.385 TWh
Solar energy	4.125 TWh
Thermal costs	849.388 Mio. €
Purchasing costs	205.256 Mio. €
Selling costs	160.178 Mio. €
Total cost	894.467 Mio. €

**Table 1** summarizes the overall deterministic results of the studied system, without considering the uncertainty of the input data. Of the total energy consumption, 7.5% is accounted for by the hydraulic power plants and 87% by the thermal power plants. The new combined cycle power plants have the lowest variable costs and are therefore utilized at 100 percent. The steam power plants are used according to their marginal costs. The gas and oil power plants used to cover peak loads have low utilization factors due to their high marginal costs. With the described model, about 10% of the electrical energy generated each year is traded on the market. A small part of the energy production with 5.5 % refers to renewable wind and solar energy.



(a)



(b)

**Fig. 3.** Scenario tree with four sections for load forecast errors (a) Symmetrical (b) Unsymmetrical. Where (n) Scenario section number, x% probability of the scenario section, yyyy number of hours in the considered scenario section and [%] Percentage deviation in the considered uncertain variable.

## 6.1. Stochastic Load

The load forecast deviations were assumed to be symmetrical and normally distributed by 10% upwards and -10% downwards. For example, the future load developments are therefore represented by the scenario tree shown in **Fig. 3-a**. Each scenario section of the tree weighted with its probability of occurrence, while the first and third scenario sections are within time range of 720 h and 2160 h, where the second and fifth scenario sections are in the stochastic time range with 2160 h and 5880 h per year, respectively. In some utilities, the input variables have unsymmetrical distribution as given in **Fig. 3-b**.

In the same way, the other input variables are processed using symmetrical or unsymmetrical scenarios trees according to their degree of uncertainties. From **Table 2**, the total costs will be increased more with the increased deviation of all input variables due to their uncertainties. This can be explained by purchasing from the available market or committing too more expensive power plants to handle these uncertainties. For the most effective variables on generation planning of the studied system, the expected value of the generation cost and its range are given in **Table 2** for symmetrical and unsymmetrical scenarios trees.

For load uncertainties of 10%, **Table 2** shows the estimated values of total expected cost and its range of deviation to be 906.387 Mio. €, 15.548% and 927.449 Mio. €, 7.517% for symmetrical and unsymmetrical scenarios trees, respectively. A similar effect can be observed in further calculations with gas price deviation. The increase in gas price leads to even higher total costs, increase in traded amounts of energy, decrease in the generation of gas-fired power plants. Variant calculation with uncertain natural gas prices of 10% leads to total expected cost and its range of deviation to be 907.286 Mio. €, 8.923% and 915.599 Mio. €, 4.4% for symmetrical and unsymmetrical scenarios trees, respectively. It is noted that the effect of the uncertainties in the spot price and water flow on the generation cost and its range of deviation is less than the effect of the uncertainties in electric load and gas price as shown in **Table 2**.

**Table 2.** Impact of input variables uncertainties on the generation costs and its scatter range.

Input variable uncertainty	Total cost Mio. € for Symmetrical tree	Total cost Mio. € for Unsymmetrical tree
Load (10%)	906.387 15.548%	927.449 7.517%
Spot price (15%)	898.538 1.335%	902.188 0.502%
Gas price (10%)	907.286 8.923%	915.599 4.4%
Water flow (8%)	904.439 1.365%	906.568 0.679%
Uncertainties of the above 4 input variables	923.982 23.374%	943.893 11.202%

## 6.2. Uncertainty in Renewable Energy Generation

With high penetration of wind energy into the generation system, the accurate forecast of wind speed is essential for generation planning. Wind speed can rapidly change, and its variation

depends on several factors, such as the surface and the local weather. In this paper, the model for characterizing the wind power is a cubic function of the wind speed. Therefore, a small error in the prediction of wind speed leads to huge variations in the wind energy estimation. To describe the range of wind speed in a particular statistical interval, the probability density function is used. In this paper the primary focus is on the normal (or Gaussian) distribution. The confidence intervals were created through historical wind speed data by examining the shapes of the respective distributions. For 90% confidence interval band with normal distribution, the standard deviation is 25.32% while for 95% confidence interval it reaches 35% [8]. With a stochastic optimization with wind speed standard deviation of 35%, the obtained results show an expected increase in the generation cost of less than 0.5%. As expected, the relative cost spread increases with increasing forecast error up to 1.9%. The reason for this lies in the relatively small wind energy feed-in of 4385 GWh per year with lower installed wind turbine capacities compared to the thermal system. With a 15% standard deviation of the wind speed, there is only a relatively slight increase in the expected value of the generation cost by the stochastic optimization. With increasing installed wind turbine power, the advantages of the stochastic approach will be increased due to the increase in rotating power capacity. This attributed to the fact that the scenario tree contains overestimates of wind energy feed-in, which may result in purchase restrictions. If then the actual wind energy feed-in is below the expected value additional generation capacity in steam power plant blocks is needed to cover the load and then the generation cost will be increased.

The comparison of the proposed stochastic optimization with a deterministic calculation considering the uncertainty in the input variable of solar radiation shows an increase of the expected generation cost by 0.42 % with a scatter range of 1.74%. It is worth notable that the amount of annual solar production is estimated to be 4125 GWh. The total costs and the traded amount of energy of the different scenario sections change similarly but to a small extent compared to the load and gas price uncertainty. This is attributed to the relatively small solar generation with lower installed solar panels capacities compared to the thermal system.

If less PV power than forecasted is obtained, more generators are committed to compensate for the power deficit and then the reserve power is reallocated according to the deviations in the PV generation. If the pre-allocated operating reserve power is unable to handle the PV uncertainty, then more gas turbines will be committed to provide more effective reserve power to compensate the lacking in PV power. When the pre-allocated reserve is enough to handle the PV uncertainty, then the available reserve is reduced by the amount of PV deficit.

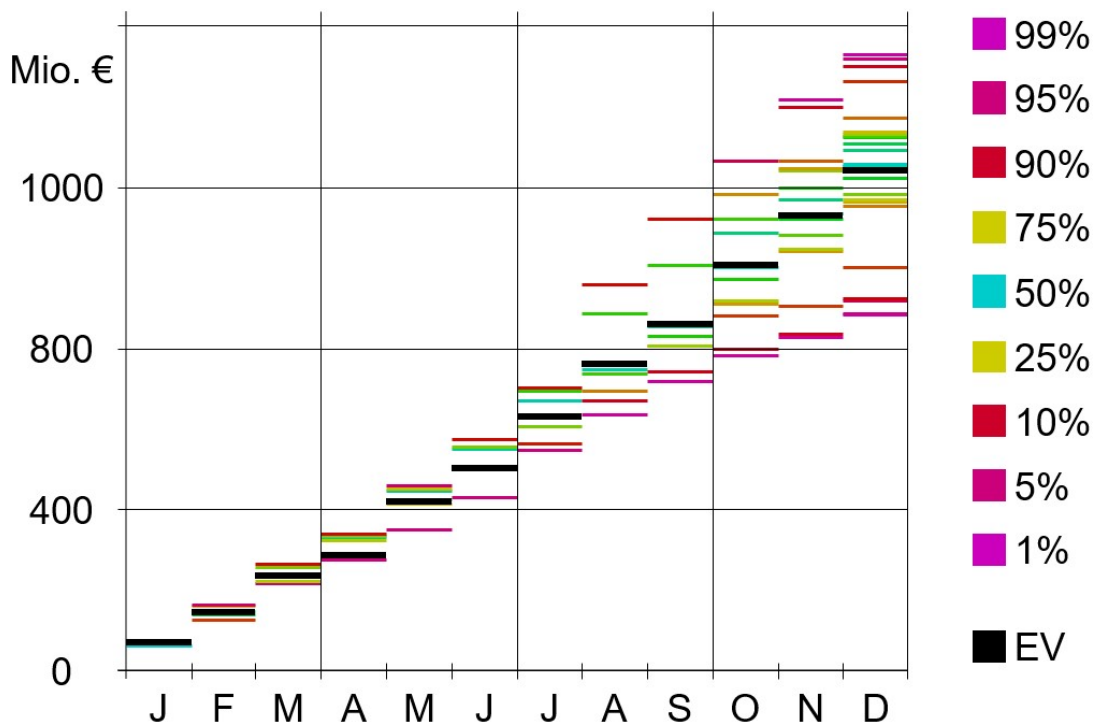
### **6.3. Influence of Scenario Tree Size**

To limit the size of the resulting scenario tree, the branching is limited only to those scenario sections whose probability exceeds a predetermined limit value. This means that the number of scenarios considered has been increased until adding further scenarios lead to an insignificant change in the generation costs. To define the effect of scenario tree size, three calculations are carried out for 4, 13 and 40 scenario sections.

**Table 3.** Effect of scenario tree size on the generation cost and its scatter range

Number of scenario sections	4	13	40
Expected generation costs (Mio. €).	902.679	913.982	914.324
Scattering rang	14.477%	23.374%	23.481%

The calculations show that the increase in the generation costs because of varying the number of scenario sections is relatively small. The reason for this is that as the number of scenario sections increase, the probability of their occurrence decreases. On the contrary, the scatter range of generation cost increases with the number of considered sections. From **Table 3** the consideration of 40 scenario sections results in very small changes in the generation cost and its range of variation compared to the 13 sections. Therefore, a scenario tree with 13 sections is sufficient to carry out the stochastic optimization for the studied system with acceptable accuracy. With simultaneous consideration of all uncertain input variables, **Fig. 4** shows the statistical development of generation cost and its scatter range along a period of one year using 13 scenario sections starting by January. This figure has thus shown that for the studied system, the cumulative generation cost steadily increases according to the increase of the required generated energy and its scattering range widens with narrow band at start but with large band in December. This is attributed to the fact that the uncertainty in input variables steadily increase over the considered planning horizon of one year.



**Fig. 4.** Statistical generation cost and its scatter range with input variable uncertainties

## CONCLUSIONS

Deterministic generation planning could cause a lot of cost pressure on electric utilities when future scenarios differ from planned deterministic scenario. This is attributed to the fact that the input variables forecast has either led to over-investment or under-investment. Consequently, the planning should be carried out considering the uncertainty of influencing variables such as the forecast of electrical load, the natural gas price, the spot market, and the wind, or solar feed-in. In addition, the generation companies must utilize their power plants with the market to optimize electricity generation and trading in an integrated manner, which represents a very computational task. Therefore, stochastic optimization and decomposition approach are proposed to effectively handle uncertainty in generation expansion planning. For this purpose, the scenario analysis approach is chosen in this study, in which the future developments of these input data are represented by scenario tree sections, which are weighted by their probability of occurrence. The stochastic optimization for large-scale system utilized the decomposition of the Lagrange relaxation to coordinate the interaction between the different subtasks, which are solved using the methods of dynamic programming and quadratic programming.

The obtained results show that the load forecast errors lead to an increase in the total generation costs. This increase grows with the mean forecast error. Underestimation and overestimation of the load do not cancel each other out due to the non-linear system behavior. The spread of the generation cost increases significantly with forecast standard deviations. A stochastic consideration of the spot prices leads also to an increase in the generation cost. Compared to the load forecast error, in this case the expected value of the generation cost changes only slightly with an uncertain spot price due to the small amounts of current energy traded. Therefore, the additional information about the forecast accuracy of the market prices is mainly of importance if the company under consideration trades a significant part of its generation on the electricity market. The stochastic consideration of the uncertain natural gas price results in an increase in the expected value of the generation cost and its spread. The deviation in natural gas prices and the market price of electrical energy leads to different use of thermal power plants and changed amounts of energy traded on the electricity market. It shows that there is a decrease in sales and an increase in purchases on the spot market with increasing natural gas price uncertainty.

Taking the uncertainties of the hydraulic supply into account primarily affects the amounts of energy traded on the spot market. Hydraulic supply uncertainty leads to a small increase in the generation cost with a clear range of variation. This then means that when carrying out planning calculations for a hydraulically dominated system, the inflow uncertainty should be considered. The test results for renewable energy supply are highly system dependent. The advantage of a stochastic approach is very small in the studied system. The reason for this lies in the small proportion of the hydraulic output in the total energy generation. With increasing installed renewable capacity, the advantages of the stochastic approach increase significantly. It can be stated that in the case of high forecast errors in the renewable, the magnitude of the power reserve, planning makes economic sense by generation optimization. The effect of the selected scenario tree size on the planning cost depends on the number of sections considered in this tree. However, there is no clear change in the spread of this cost range as the number of sections continues to increase. On the other side, the generation cost and its range are clearly increased with the degree of asymmetry in the scenario trees. This is because in an asymmetrical scenario tree, the higher

probability of an influencing variable occurring on one side of the uncertain areas leads to an even greater change in the use of the power plant.

All input parameters are assumed independent/uncorrelated by estimating the economic generation cost. The effect of correlated input variables needs further analysis. More research should be conducted to define suitable size of scenario tree for each input variable using RNN. Future work might explore whether Bayesian optimization can be used for generation planning under uncertain input variables. These will enhance the performance of the algorithm and help the decision making to take the proper actions.

## ACKNOWLEDGMENT

The author would like to thank Univ.-Prof. Dr.-Ing. A. Moser director of IAEW at RWTH AACHN and AvH-Stiftung for their support.

## References

- [1] S. Nießen, H.-J. Haubrich, B. Krasenbrink, "Risk Management in Power Generation and Trading," VDIReport, No. 1508, VDI-Verlag, pp. 53-62, Düsseldorf, 1999.
- [2] H. Neus, B. Stern, H.-J. Haubrich, H.-P. Flicke, "Spot market-oriented use optimization of combination blocks," *Electricity Industry*, vol. 99, H.25, 2000.
- [3] Maciej Świechowski, Konrad Godlewski, Bartosz Sawicki Jacek Mańdziuk, "Monte Carlo Tree Search: a review of recent modifications and applications", *Artificial Intelligence Review* (2023) 56:2497–2562.
- [4] Yuquan Xie, Wen Hu, Xilin Zhou, Shuting Yan and Chuancheng Li, "Artificial Neural Network Modeling for Predicting and Evaluating the Mean Radiant Temperature around Buildings on Hot Summer Days", *Buildings* 2022, 12, 513, pp 2-22.
- [5] Shiwei YU, Limin YOU, Shuangshuang ZHOU, "A review of optimization modeling and solution methods in renewable energy systems", *Front. Eng. Manag.* 2023, 10(4): 640–671.
- [6] Thanh Van Nguyen, "Electricity Generation Cost Optimization Based on Lagrange Function and Local Search", *Journal of Computing Theories and Applications*, 104-114, 2023, vol. 1, no. 2, 104-114.
- [7] Chen Wu, Zhinong Wei, Xiangchen Jiang, Yizhen Huang and Donglou Fan, "Optimization Decomposition of Monthly Contracts for Integrated Energy Service Provider Considering Spot Market Bidding Equilibria", *Electronics* 2024, 13, 1945, pp 2-22.
- [8] B. Xu, P. Zhong, R. C. Zambon, Y. Zhao, and W. W.-G. Yeh, "Scenario tree reduction in stochastic programming with resource for hydropower operations," *AGU, Water Resour. Res.*, vol. 51, 6359–6380, July 2015.
- [9] H. Neus, H. Schmöller, B. Pribicevic, H.-P. Flicke, "Integrated Optimization of Power Generation and Power Trading - Process Requirements and Practical Experience," *Annual Report 2002 of the IAEW of the RWTH Aachen, ABEV*, Vol. 86, pp. 88-92, Aachen 2002.
- [10] B.-M. Hodge, D. Lew, and M. Milligan, "Short-Term Load Forecasting Error Distributions and Implications for Renewable Integration Studies," 2013 IEEE Green Technologies Conference (GreenTech), Denver, CO, USA, pp. 435-442, 2013.
- [11] Vineet Kumar Singh, Neha Singh Chauhan, and Deepti Kushwaha "An Overview of Hydroelectric Power Plant," *ISST Journal of Mechanical Engineering*, vol. 6, no. 1, 2015.
- [12] T. Barforoushi, H. Javidi, "A New Model Considering Uncertainties for Power Market Simulation in Medium-Term Generation Planning," *Iranian Journal of Electrical and Electronic Engineering*, May 2006.
- [13] B.-M. Hodge and M. Milligan, "Wind Power Forecasting Error Distributions over Multiple Timescales," *Power & Energy Society General Meeting Detroit, Michigan July 24-29, 2011*.
- [14] T. Kim, Y. Chang, D. Kim, and M. Kim, "Preventive Maintenance and Forced Outages in Power Plants in Korea," *Energies*, vol. 13, no. 14, 3571, 2020.
- [15] H. Heitsch and W. Römisch, "Scenario reduction algorithms in stochastic programming," *Computational Optimization and Applications*, vol. 24, pp. 187–206, 2003.



- [16] R. Goncalves, E. Finardi, E. Da Silva, and M. Dos Santos, "Comparing stochastic optimization methods to solve the medium-term operation planning problem," *Comp. Appl. Math.*, vol. 30, no. 2, pp. 289–313, 2011.
- [17] J. Hou, Q. Zhai, Y. Zhou, X. Guan, "A Fast Solution Method for Large-scale Unit Commitment Based on Lagrangian Relaxation and Dynamic Programming," *Mathematics, Optimization and Control*, arXiv:2211.01601v1, Nov. 2022.
- [18] X. Yu, X. Zhang, "Unit commitment using Lagrangian relaxation and particle swarm optimization," *Int J Electr Power Energy Syst.*, vol. 61, pp. 510-522, 2014.
- [19] H. Zhao, H. Lu, B. Li, X. Wang, S. Zhang, and Y. Wang, "Stochastic Optimization of Microgrid Participating Day-Ahead Market Operation Strategy with Consideration of Energy Storage System and Demand Response," *Energies*, vol. 13, no. 5, 1255, pp. 1-16, 2020.
- [20] M. Fan, V. Vittal, G. Heydt, and R. Ayyanar, "Preprocessing Uncertain Photovoltaic Data," *IEEE Transactions on Sustainable Energy*, vol. 5, no. 1, pp. 351-352, Jan. 2014.