Journal of Al-Azhar University Engineering Sector

Vol. 19, No. 73, October 2024, 1191 - 1208

A PARAMETRIC STRATEGY FOR GEOMETRY OF ARCHITECTURAL FREEFORM STRUCTURE

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Citation:

M. A. Youssef " A Parametric Strategy for Geometry Of Architectural Freeform Structure ", Journal of Al-Azhar University Engineering Sector, vol. 19, pp. 1191 - 1208, 2024.

Received: 24 March 2024

Revised: 22 July 2024

Accepted: 31 August 2024

DOI: 10.21608/auej.2024.278976.1640

ABSTRACT

Grid paterns represent a modern structural system designed for wide coverage and, owing to their geometry, effectively withstand forces. The primary advantage of utilizing a continuous shell lies in its ability to facilitate force flow in all directions. Hence, this study will evaluate the principle stress lines to identify optimal grid pattern topology. This analysis will be conducted for comparison with geometric patterns, providing insights into structurally efficient solutions. Moreover, the study assumes steel as the primary material for grid shells, considering its widespread usage in construction. By investigating arrangements of grid shells and expanding beyond regular geometric shapes to embrace freeform architectures, the research aims to contribute to the advancement of innovative structural solutions. The effectiveness of a grid shell depends on how forces are transferred to the beams and ultimately to the ground. Therefore, the shape and orientation of the grid play a crucial role in determining structural efficiency. This study seeks to identify and optimize such configurations for practical implementation in real-world architectural projects.

KEYWORDS: Grid Geometry, Freeform Structure, Form Finding

الاستراتيجية البارامترية لهندسة الهياكل المعمارية الحرة

مهجة على عبدالسلام يوسف [1]

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الملخص

يعتبر بناء سطح حر من أجزاء أصغر هو أحد التحديات الهامة في الهندسة المعمارية، والتي يمكن معالجتها عن طريق تقسيم الأسطح غير المنتظمة إلى أقسام أصغر. تكمن إمكانيات التصميم المعماري في دراسة الأشكال وكيفية إنشاء فكرة أولية لنمط التغطية من خلال إنشاء نمط يغطي الهيكل بالكامل. يتمحور هدف هذا البحث في استكشاف سلوك أنماط مختلفة من الهياكل الشبكية، مثل الشبكة الرباعية والماسية والمثلثة والمربعة الثلاثية والشبكة السداسية. والميزة الرئيسية للأشكال الحرة المستمرة هي قدرتها على تيسير تدفق القوى في جميع الاتجاهات. سيتم النظر في القوى الضغطية الرئيسية، حيث توفر مؤشرًا على الطوبولوجيا المثلى لنمط الشبكة، مع إدراج ذلك للمقارنة مع الأنماط الهندسية. ستدرس هذه الورقة أيضًا الأمثلة الحالية من جميع أنحاء العالم. تتمحور مناقشة هذه الورقة حول دراسة الهياكل الشبكية ذات الشكل الحر، حيث يؤثر شكل الشبكة واتجاهها على تدفق القوى من خلال الهيكل، بهدف العثور على حلول هيكلية فعّالة.

الكلمات المفتاحية: الشبكات الهندسية، الهياكل الحرة الشكل، العثور على النموذج

1. INTRODUCTION

In architectural geometry, a fundamental challenge lies in constructing free-form surfaces from smaller sections. This obstacle can be addressed through panelization, where free-form surfaces are discretized into smaller pieces, or by approximating the desired surface with shapes that offer better geometric properties for construction. Grid shells represent a modern structural system designed for wide coverage and, owing to their geometry, effectively withstand forces[1] . They are recognized as the lightest, most cost-efficient, and quickest method for covering significant areas. Grid shells exhibit unique load-bearing behavior as they resist loads through the axial force of the lattice [2] .

2. OBJECTIVE

This grid pattern analysis aims to investigate how grid pattern behavior varies depending on shape, with a focus on expanding beyond regular geometric forms to explore freeform designs within structured grid shells. In grid shell structures, forces are conveyed through beams to the ground, with the grid's shape and orientation crucial in determining the efficiency of force distribution within the structure. The objective of this research is to explore the potential of architectural design in generating façade patterns that encompass entire building structures, with a focus on investigating the behavior of various designs of reticulated bars.

3. LITERATURE REVIEW

Many studies have reported in this area to investigate the applications of grid pattern in architecture, many of these researches are only oriented to rigid and elastic grid pattern with specific meshes. a new fundamental key for the design and erection of rigid and elastic gridshells with specific meshes. For interlaces with uniqueness are shaped based on the exceptional geometry of a previous successful project designed in 2011 by the Navier Laboratory of the École des Points et Chaussées [3].

Other researches have pointed at the behavior of materials. However, topics related to the relationship between shape and structural behaviour of grid-shells on the architectural form and its contribution to find structurally efficient solutions, and the participation of this Structural Analysis in the early stages of the design process are not yet established.

4. METHODOLGY

In order to achieve the mentioned objective, the methodology implemented in this paper to find structurally efficient solutions. It will explore various designs of reticulated bars, including quadrilateral, diamond, triangular, triangular square, and hexagonal gridshells. The primary aim is to capitalize on the inherent advantages of continuous shells, particularly in facilitating force flow in all directions. Principle stress lines will be evaluated to indicate optimal grid pattern topology, primarily for comparison with geometric patterns rather than direct modeling or implementation. The research will also examine existing global instances of grid shell structures. Each freeform grid pattern is estimated to cover an area of 100 m2, with steel assumed as the primary material for the panels. Computational and digital modeling, utilizing software such as Rhinoceros, Grasshopper, LunchBox, and Karamba, will be employed to compare and analyze the grids of interest.

5. DESIGN ASPECTS OF PATTERNS FOR GRID SHELL

A pattern's topology and geometry are its two paramount design aspects [4].

5.1 Geometrical aspects

The arrangement of vertices, specified by their coordinates (x, y, z) in three-dimensional space, defines a pattern's geometry. In designs with n vertices, there exist degrees of geometric freedom, allowing for continuous design and optimization methods. Organizing these parameters can add coherence and constraints to the design space, ultimately reducing the search area's size.

5.2 Topological aspects

In this study, a pattern's topology refers to the interconnectedness of its vertices, constituting a complex combinatorial challenge involving varying discrete factors. Conversely, geometric parameters entail a specified set of continuous variables **Fig.1**. The topology is inherently intertwined with the geometry; hence, two patterns sharing identical geometry must exhibit identical topology. However, two patterns with matching topology may not necessarily share the same geometry. When two patterns undergo a continuous and reversible geometrical transformation known as homotopy, they retain the same topology without altering the shape or disrupting the pattern. For instance, a square and a rectangle possess the same topology but differ in geometry, while a disc and a sphere exhibit neither identical topology nor geometry. [5].

Fig. 1. Different grid patterns for grid shell

Source; the researcher according to Hojjat, M., Stavropoulou view

6. GRID SHELL CLASSIFICATION

Grid shells represent a versatile structural form with numerous variations. They are characterized by curved surfaces with an exceptionally low depth-to-span ratio. Classification of grid shells can be based on their geometry, method of load transmission, shape, and structure.

6.1 Grids geometry basic patterns

Different geometric modules can be used to create a grid shell. However, square, diamond, triangular, triangulated squares, and hexagonal grid are the most commonly utilised modules as fundamental patterns **Fig. 2**. [6].

Fig. 2. Basic patterns for grid geometry.
Source; the researcher according to Chilton, *Space Grid Structures*.

In essential challenge in architectural geometry revolves around constructing free-form surfaces from smaller sections. This hurdle can be overcome by either discretizing the free-form surfaces into smaller pieces, a process referred to as panelization, or by seeking an approximation of the desired surface with improved geometric properties, particularly from a construction standpoint [7].

7. FORM FINDING

The concept of form finding can be summarized as "Form Follows Force," where the structural form is dictated by the forces it experiences to ensure effective loads management. Surface stressed structures are typically categorized as either tensile or compression structures. The procedure outlined in this study for constructing a grid shell structure is illustrated in **Fig.3**, comprising three main steps: Geometry, Form Finding, and Optimization [8].

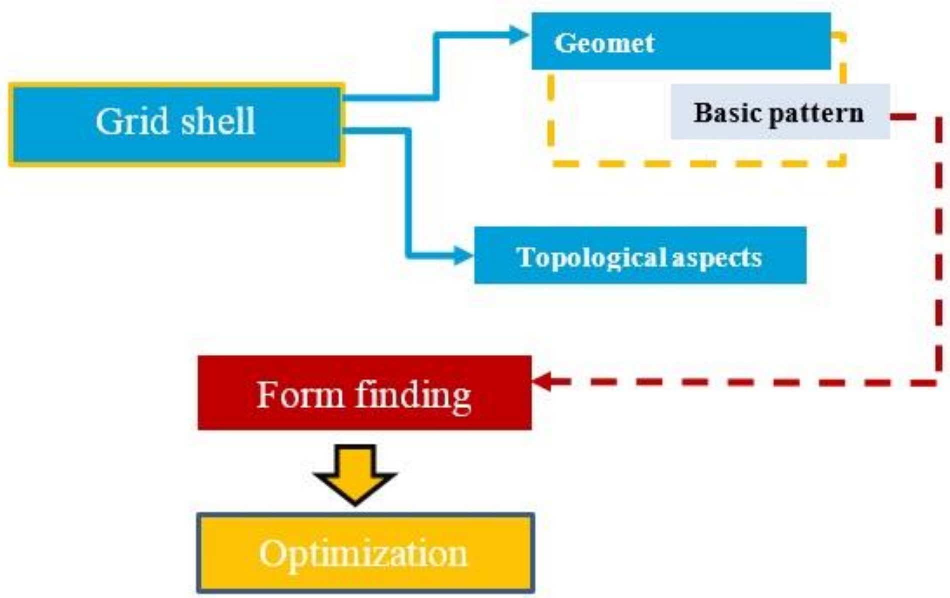


Fig. 3. Flow chart of the process used in this study to form find a grid shell structure

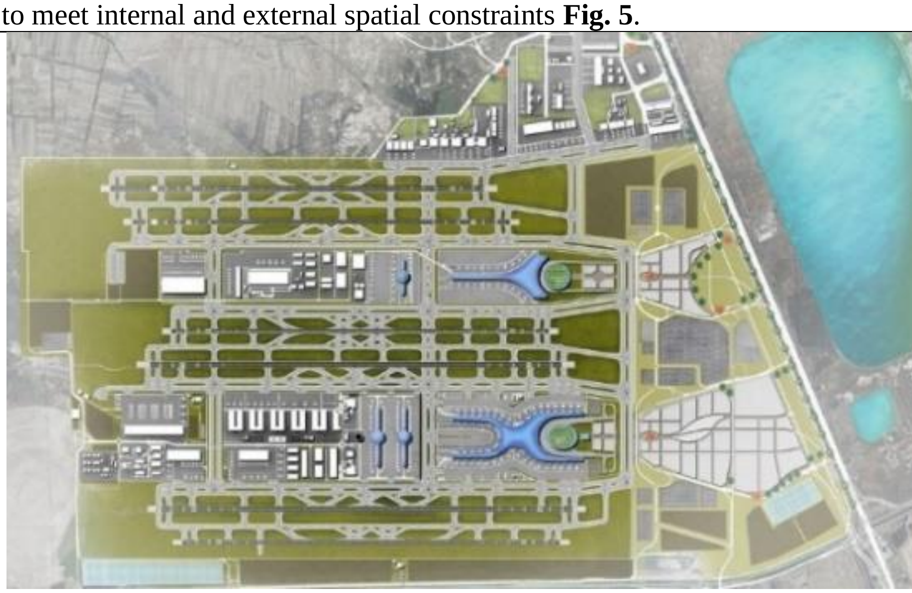
Source; the researcher according to Ochsendorf & Block.

7.1. Triangular Gridshell

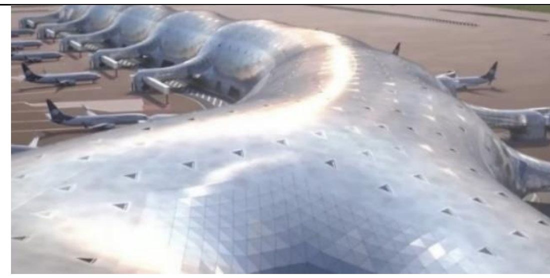
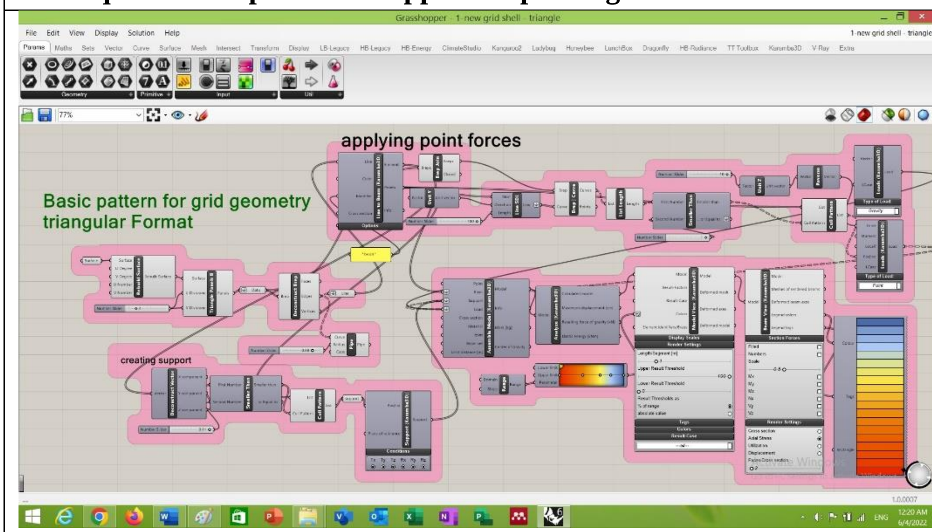
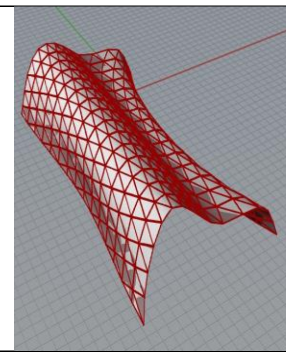
Surface triangulation, being a common approach to discretization, is known for its advantage in consistently producing flat panels between three points, allowing any given surface to be resembled by a planar triangular grid.

The large number of elements and high node complexity in a triangular Gridshell are downsides, as each node typically has six edges. It's often noted that these drawbacks lead to limited structural transparency, which becomes problematic when working with see-through covering materials like glass [9].

7.1.1. Airport in Mexico City

<table>
<tr><td colspan="3">Triangular format geometry</td></tr>
<tr><td colspan="3">Project: Airport in Mexico City[10]</td></tr>
<tr><td rowspan="5"></td><td>Architects: Foster + Partners</td><td rowspan="5"></td></tr>
<tr><td>Location: Mexico City</td></tr>
<tr><td>Year: 2020</td></tr>
<tr><td>Area: 288,000 m^2</td></tr>
<tr><td>Lobel Patterns –
of equilateral triangles</td></tr>
<tr><td colspan="3">Description:</td></tr>
<tr><td colspan="3">The airport, recognized as one of the world's largest, showcases a smoothly contoured ceiling enveloping the terminal, covering an approximate area of 500,000 square meters. Its roof is structured with a two-layer triangulated space frame, boasting maximum spans of 170 meters. Vertical loads are efficiently channeled along the perimeter and through vertical supports (funnels), integral elements of the space frame geometry Fig. 4. To optimize structural integrity, the design follows a catenary curve, with minor adjustments to meet internal and external spatial constraints Fig. 5.</td></tr>
<tr><td colspan="3">
Fig.4. Master Plan -general layout of the facilities within the new site.</td></tr>
</table>

7.1.2. The Equation Script - Grasshopper Script using Karamba 3D

<table>
<tr><td colspan="3">Triangular format geometry</td></tr>
<tr><td colspan="3">Airport in Mexico City</td></tr>
<tr><td colspan="3"> 
Fig.5. MCA overview and interior visualization
(Image: Foster + Partners).</td></tr>
<tr><td colspan="3">The Equation Script - Grasshopper Script using Karamba 3D</td></tr>
<tr><td colspan="2" rowspan="6">
</td><td></td></tr>
<tr><td>Span 7mx15m</td></tr>
<tr><td>Displacement x, y = 0.5m</td></tr>
<tr><td>Lath size 40mm</td></tr>
<tr><td>Material Steel</td></tr>
<tr><td>Weight = 1993 kg</td></tr>
<tr><td rowspan="2">Load Case 1: Grids basic patterns- Triangle</td><td colspan="2">Normal force Tension = 2.75 e+1 KN</td></tr>
<tr><td colspan="2">Normal force Compression = -6.42 e+1 KN</td></tr>
</table>

7.2 Quadrilateral Gridshell

A quadrilateral grid offers various advantages over a triangle mesh. It has fewer edges, potentially resulting in less use of structural material, simplified node complexity, and improved structural transparency. Schober suggests that quadrilateral grids are notably simpler to manufacture than triangle grids due to their reduced need for mullions and machining operations.

7.2.1 Chadstone shopping center – Melbourne, 2016

<table>
<tr><td colspan="2">Square format geometry</td></tr>
<tr><td colspan="2">Project: Chadstone shopping center – Melbourne [11]</td></tr>
<tr><td rowspan="5"></td><td>Architect: CallisonRTKL</td></tr>
<tr><td>Location: Victoria, Australia</td></tr>
<tr><td>Year: 2016</td></tr>
<tr><td>Area: 129,924 m²</td></tr>
<tr><td>Grids geometry
Basic patterns - Square</td></tr>
<tr><td colspan="2">Description: [12]</td></tr>
<tr><td colspan="2">The Chadstone Shopping Centre in southeastern Melbourne has regained its status as Australia's largest following extensive renovations in 2017. One of the standout features of the redevelopment is the impressive 31-meter-high, 7,000-square-meter gridshell glass roof, creating spacious, column-free interiors. Fig.6. Developed in collaboration with engineering experts and research departments from the Universities of Stuttgart and Bath, the design utilized advanced 3D parametric modeling techniques and was further enhanced with a mix of imagery and video. Notably, this innovative roof represents a first of its kind on the continent. Fig.7..</td></tr>
<tr><td colspan="2">
Fig.6. layout</td></tr>
</table>

7.2.2 The Equation Script - Grasshopper Script using Karamba 3D

Square format geometry

Chadstone shopping center – Melbourne [13]



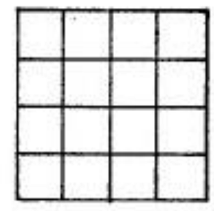
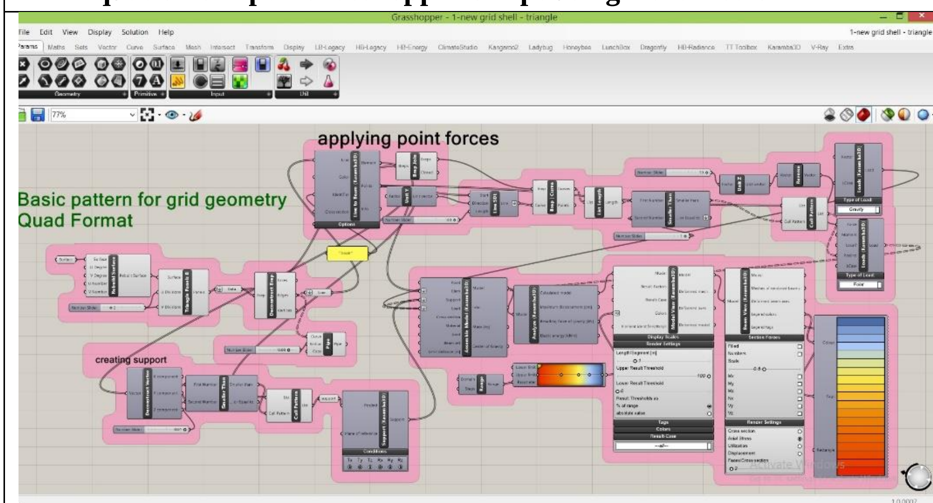
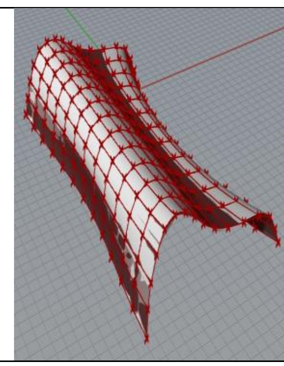


Fig.7. The glass roof

The Equation Script - Grasshopper Script using Karamba 3D





Span 6mx9m
Displacement x, y = 0.5m
Lath size 40mm
Material Steel
Weight = 1557 kg

Load Case 2: Grids geometry basic patterns- **Square**	Normal force Tension = 2.4 e^{+1} KN
	Normal force Compression = -11.9 e^{+1} KN

7.3. Diamond Gridshell

7.3.1. Harbin Opera House

<table>
<tr><td colspan="2">Diamond format geometry</td></tr>
<tr><td colspan="2">Project: Harbin Opera House [15][14]</td></tr>
<tr><td rowspan="5"></td><td>Architect: MAD Architects</td></tr>
<tr><td>Location: Harbin, China</td></tr>
<tr><td>Year: 2015</td></tr>
<tr><td>Area: 850000 ft²</td></tr>
<tr><td>Grids geometry
basic patterns - Diamond</td></tr>
<tr><td colspan="2">Description: [16]</td></tr>
<tr><td colspan="2">Nestled within Harbin’s wetlands, the Harbin Opera House was conceived in harmony with the rugged landscape and harsh climate of the northern city. Its design, reminiscent of natural forces like wind and water, seamlessly merges with the surroundings, embodying the essence of local identity, art, and culture Fig.8.. . Ma Yansong, founding principal of MAD Architects, envisions the opera house as a beacon of cultural innovation—a versatile performance venue and a striking public space that reflects the fusion of human expression, artistic vision, and urban identity, all while harmonizing with the natural environment Fig.9..</td></tr>
<tr><td colspan="2">
Fig. 8. Harbin layout</td></tr>
</table>

7.3.2. The Equation Script - Grasshopper Script using Karamba 3D

Diamond format geometry
Harbin Opera House

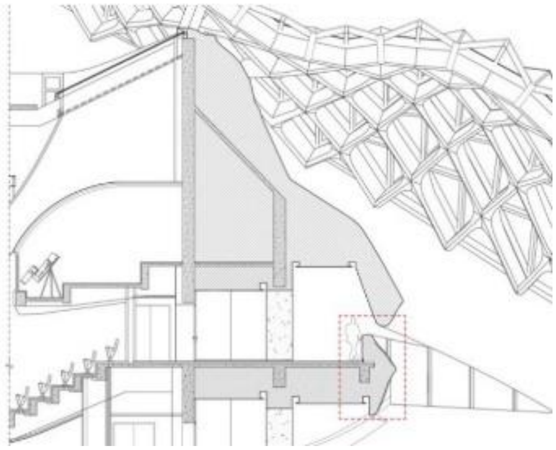
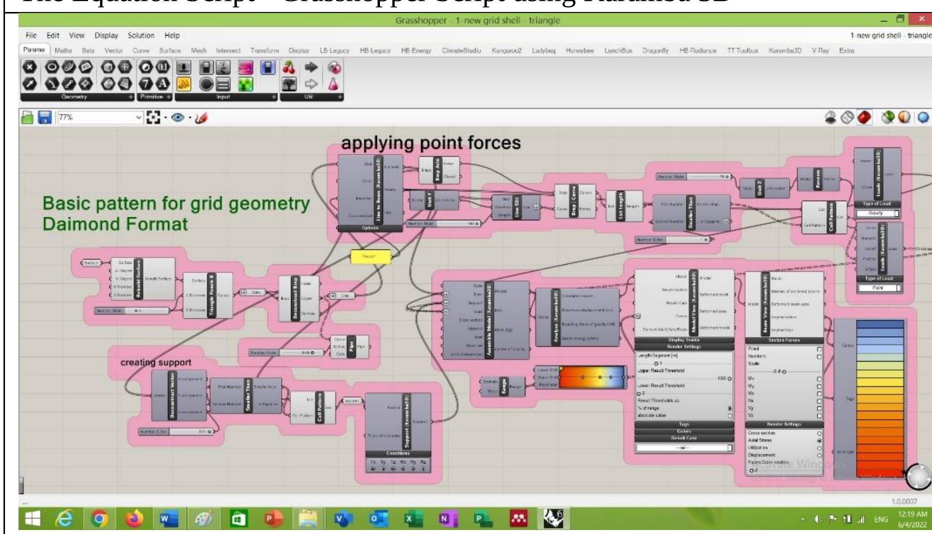
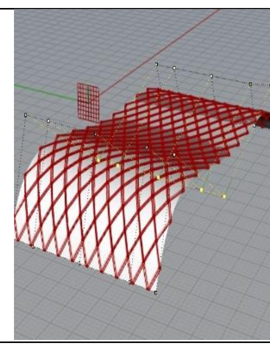


Fig. 9. Section detail Harbin Opera House

The Equation Script - Grasshopper Script using Karamba 3D





Span 6mx9m	
Displacement x, y = 0.5m	
Lath size 40mm	
Material Steel	
Weight = 1431 kg	
Load Case 3: Grids basic patterns- **Diamond**	Normal force Tension = 70.2 e+1 KN
	Normal force Compression = -72.3 e+1 KN

7.4. Triangulated Square Gridshell

7.4.1 New Milan trade fair

Triangulated square Format geometry		
Project: New Milan trade fair		
	**Architects:** Massimiliano , Fuksas **Location:** Milan,Italy **Year:** 2005 **Area:** 46,300 m² **Grids geometry** **Triangulated square**	
[redacted]		
Description: [17]		
The massive glass and steel framework of the Milan Fair incorporates a combination of quadrilateral and triangular panels, as depicted in **Fig.10**. Initially, the surface was designed as a triangular mesh but was later transformed into a quadrilateral one. Thinner rods were added in areas where elements were not planar, resulting in the creation of planar triangles, as depicted in **Fig.11.** This optimization helps to minimize the use of material in non-essential areas.		
 Fig. 10. site plan **Source;** https://www.area-arch.it/en/milan-memoryinvention-nuovo-polo-fiera-milano/ Retrieved May 2021		

7.4.2 The Equation Script - Grasshopper Script using Karamba 3D

Triangulated square Format geometry

New Milan trade fair

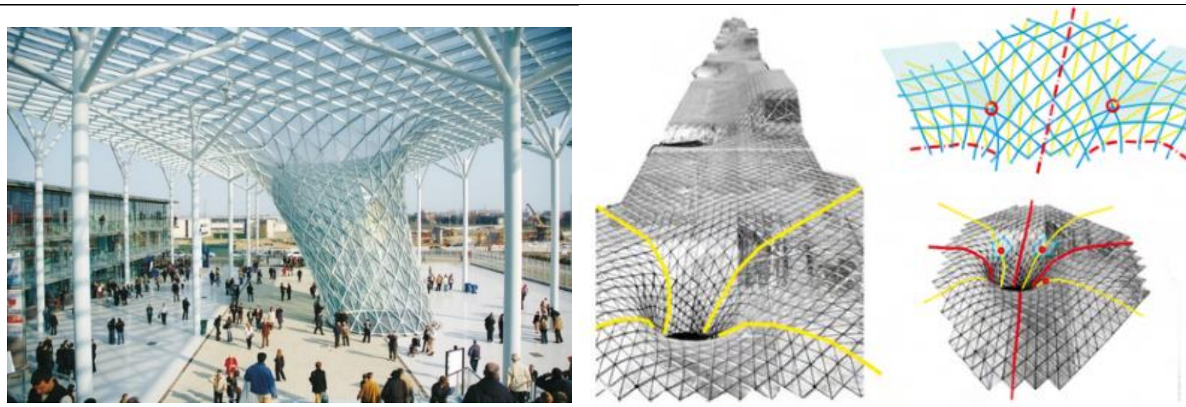
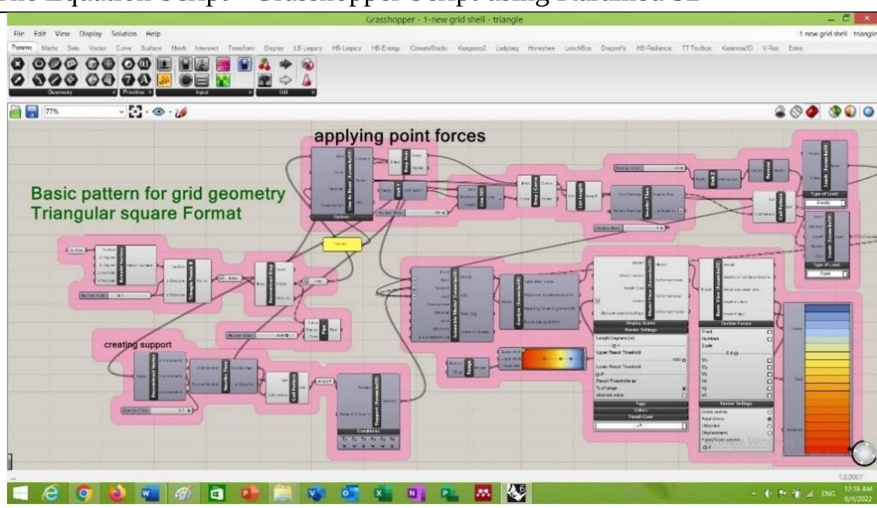
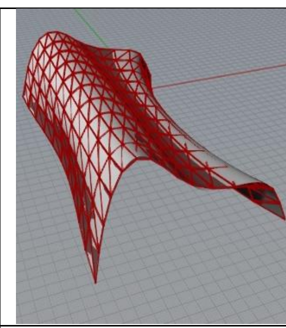


Fig. 11. The New Milan Trade Fair, Location where triangular panels transition into quadrilateral panels.

The Equation Script - Grasshopper Script using Karamba 3D





Span 6mx9m

Displacement x, y = 0.5m

Lath size 40mm

Material Steel

Weight = 2284 kg

Load Case 4: Grids geometry **- Triangulated square**	Normal force Tension = 70 e+1 KN
	Normal force Compression = -79 e+1 KN

7.5 Hexagonal Gridshell

A hexagonal grid provides an alternative approach to discretizing a surface. Due to the significance of node complexity in manufacturing costs, a hexagonal grid offers several advantages over a triangular or quadrilateral mesh, as it typically involves only three beams linked at each node. Additionally, hexagonal grids often possess aesthetic appeal, resembling organic structures in some cases. The Honeycomb subdivision procedure facilitates this transition by converting a triangular mesh into a hex-dominant one.

While planar hexagonal meshes may be well-suited for certain shapes, such as those with constant mean curvature, applying such meshes to arbitrary freeform surfaces can prove exceedingly challenging, and in some cases, impossible, without causing self-intersections. In such situations, alternative geometric shapes may be required to enable the creation of a planar hexagonal mesh that effectively covers the surface.[18] .

7.5.1. SUTD Library Pavilion / City Form Lab

<table>
<tr><td colspan="3">Hexagonal Format geometry</td></tr>
<tr><td colspan="3">Project: SUTD Library Pavilion / City Form Lab [19]</td></tr>
<tr><td rowspan="6"></td><td>Architects: City Form Lab</td><td rowspan="6"></td></tr>
<tr><td>Location: Singapore</td></tr>
<tr><td>Year: 2013</td></tr>
<tr><td>Area: 300 m²</td></tr>
<tr><td>Grids geometry</td></tr>
<tr><td>basic patterns - Hexagon</td></tr>
<tr><td colspan="3">Description:</td></tr>
<tr><td colspan="3">Situated on a sloping lawn at the temporary Dover Campus, the library pavilion of the Singapore University of Technology and Design (SUTD) serves as a shaded open-air space for university students and staff to relax, work, and mingle during the day Fig.12. As night descends, it transforms into a venue for informal gatherings, evening talks, and SUTD community activities Fig.13.

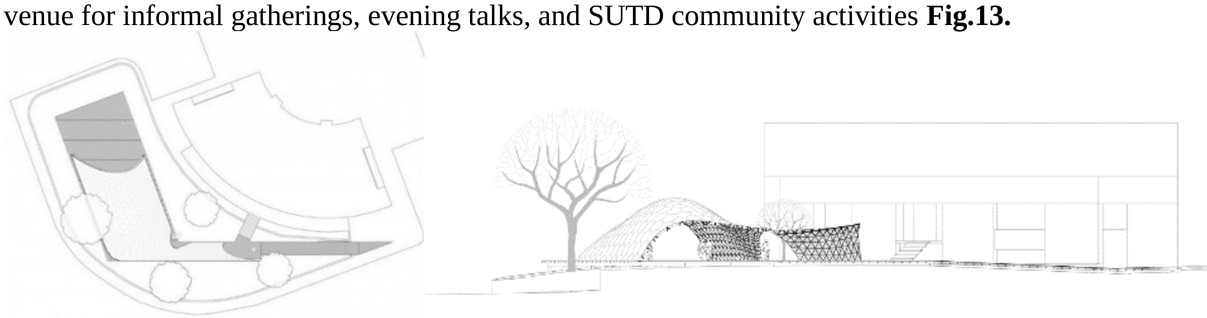
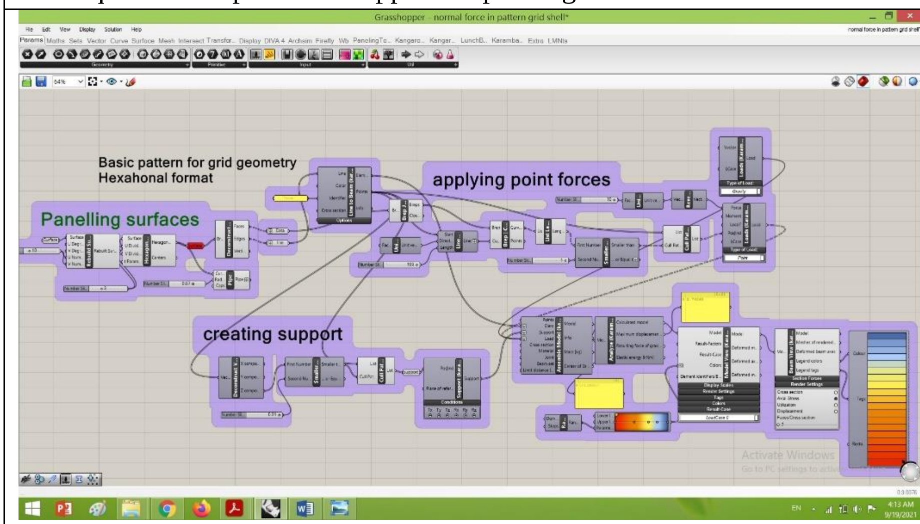
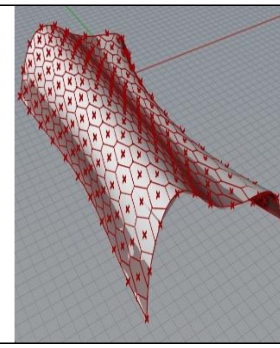



Fig.12. The pavilion utilizes a catenary structure.</td></tr>
</table>

7.5.2 The Equation Script - Grasshopper Script using Karamba 3D

<table>
<tr><td colspan="3">Hexagonal Format geometry</td></tr>
<tr><td colspan="3">SUTD Library Pavilion / City Form Lab</td></tr>
<tr><td colspan="3">
Fig.13. The form reduces the requirement for columns, beams, and vertical walls.</td></tr>
<tr><td colspan="3">The Equation Script - Grasshopper Script using Karamba 3D</td></tr>
<tr><td colspan="2" rowspan="6">
</td><td></td></tr>
<tr><td>Span 6mx9m</td></tr>
<tr><td>Displacement x, y = 0.5m</td></tr>
<tr><td>Lath size 40mm</td></tr>
<tr><td>Material Steel</td></tr>
<tr><td>Weight = 1198 kg</td></tr>
<tr><td rowspan="2">Load Case 5: Grids basic patterns- Hexagon</td><td colspan="2">Normal force Tension = 72 e+1 KN</td></tr>
<tr><td colspan="2">Normal force Compression = -74.7 e+1 KN</td></tr>
</table>

8. Base case simulation results

Presented in **Fig.14**, the normal forces occurring in the grid shells are illustrated. The extreme values of the grid, Ncompression and Ntension, are depicted, with compression (blue) represented as negative and tension (orange) as positive. The Triangulated square pattern displays the highest compression force, while the hexagonal grid shows the strongest tension force. Additionally, the square and triangular patterns exhibit the least tension and compression forces, resulting in the narrowest range between maximum compression and tension. **Table 2**. offers additional insights.

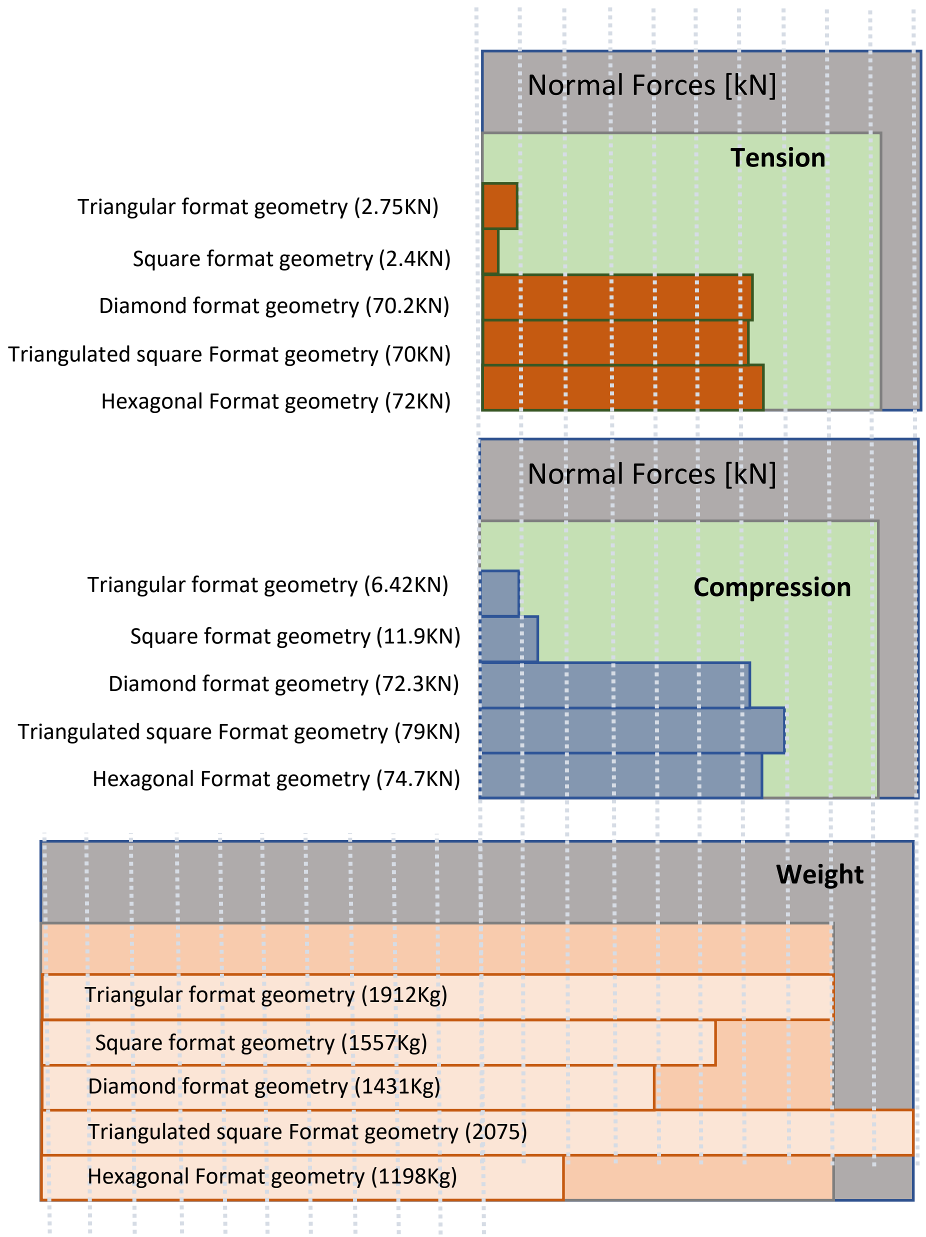


Fig. 14. Normal forces [kN] in the grid shell for different grid patterns

Source; by the researcher

Table 2. Grid pattern analysis

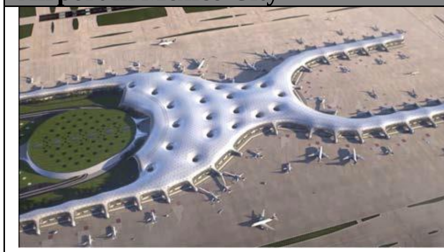
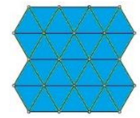
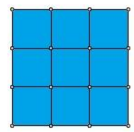
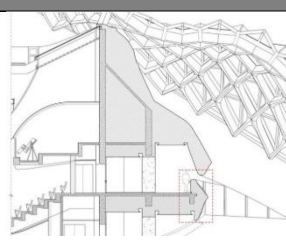
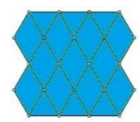
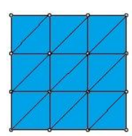
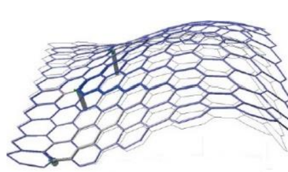
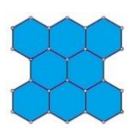
Triangular Format geometry			
Airport in Mexico City			
			Span 6mx18m Displacement x, y = 0.5m Lath size 40mm Material Steel Weight = 1993 kg Normal force Tension = 2.75 e+1 KN Normal force Compression = -6.42 e+1 KN
Quad Format geometry			
Chadstone shopping center			
			Span 6mx18m Displacement x, y = 0.5m Lath size 40mm Material Steel Weight = 1557 kg Normal force Tension = 2.4 e+1 KN Normal force Compression = -11.9 e+1 KN
Diamond Format geometry			
Harbin Opera House			
			Span 6mx18m Displacement x, y = 0.5m Lath size 40mm Material Steel Weight = 1431 kg Normal force Tension = 70.2 e+1 KN Normal force Compression = -72.3 e+1 KN
Triangulated Square Format geometry			
New Trade Fair Milan			
			Span 6mx18m Displacement x, y = 0.5m Lath size 40mm Material Steel Weight = 2284 kg Normal force Tension = 70 e+1 KN Normal force Compression = -79 e+1 KN
Hexagonal Format geometry			
SUTD Library Pavilion / City Form Lab			
			Span 6mx18m Displacement x, y = 0.5m Lath size 40mm Material Steel Weight = 1198 kg Normal force Tension = 72 e+1 KN Normal force Compression = -74.7 e+1 KN

Table 3. Grid pattern analysis

Triangular Format geometry	There are some disadvantages that we have to consider: In a steel/glass or other construction based on a triangle mesh, six beams meet in a node; this inplies a higher node complexity. The cost of triangular glass panels are higher per-area than the cost of quadrilateral panels. More nodes imply more steel and glass, and as a consequence more weight. Apart from simple cases triangle meshes do not possess offsets at constant face-face or edge-edge distance. Triangle meshes have high valence as geometric torsion on the nodes.
Quad Format geometry	Quadrilateral grid exhibit two remarkable disadvantages: on the one hand their stiffness is lower and on the other hand we have to consider non-planar panels in general. Flat panels are of course cheap to produce, but also single curvature panels can be obtained at little cost through the cold bending technique.
Hexagonal Format geometry	Similar as for a quadrilateral mesh, the hexagonal mesh also needs either connections that can transfer moment or additional structurally stabilizing members in order to be structurally stable. However, not all patterns provide desired properties in for example how well they are able to approximate doubly curved surfaces. Honeycomb patterns has shown to be a good representation of arbitrary surfaces when aiming for repetition within the pattern. According to Pottman et al. an arbitrary, smooth surface can be approximated with a non-smooth grid pattern using the honeycomb figuration

Conclusions

By employing form-finding computational approaches, architects have been able to seamlessly integrate their design concepts with nature, gaining deeper insights into the structural aesthetic requirements of their projects. The proposed conceptual design tools are built on 'geometry' for the computational form discovery process. To develop real and virtual shapes, architects employed computer modelling and simulation. As a result, computational patterns and parametric models were employed to produce generative designs based on geometric relationships. To summarise, the shape and direction of the grid impact how efficiently forces flow through the structure; this process seeks structurally efficient solutions.

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